Senior 4 Pre-Calculus Mathematics (40S) Standards Test

Administration Manual for Supervising Teachers

January 2004

Manitoba

Building for the Future

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Manitoba Education, Citizenship and Youth,
Assessment and Evaluation Branch,
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Preface

This *Administration Manual for Supervising Teachers* is distributed to teachers and administrators, and must be read prior to administering the standards test. The rules and guidelines in this manual must be strictly observed to ensure the consistency of standards test administration.

Please use this manual in conjunction with the registration package, the *Policies and Procedures for Standards Tests* handbook and the *Information Bulletin* (these documents are available on our website at: http://www.edu.gov.mb.ca/ks4/assess/publications.html) to gain a full understanding of the protocols and procedures associated with this test. This manual contains general information about the test, step-by-step instructions for the test administration, and policies regarding absenteeism, lateness, violation of rules, etc.

If you encounter exceptional circumstances not addressed in this manual, or if you have further inquiries, contact Carol Chabbert, Administrative Officer, at 945-0747 or toll-free at 1 800 282-8069, extension 0747.

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General Information

Manitoba Education, Citizenship and Youth develops all standards tests in collaboration with teachers from across the province. This standards test is based on *Senior 4 Mathematics Manitoba Curriculum Framework of Outcomes and Senior 4 Standards (2001).*

The standards test is locally marked and counts for 30% of each student's final grade in the course. Student results on the test will be released to students according to local policy on the date designated by local jurisdictions.

All students enrolled in Senior 4 Pre-Calculus Mathematics (40S), except those who have been exempted, must write the Senior 4 Pre-Calculus Mathematics (40S) Standards Test on January 20, 2004. However, a student may rewrite a mandatory provincial standards test at any regularly scheduled administration without being registered in the course (see *Policies and Procedures for Standards Tests*, Section 3.1).

Marking instructions are provided by the department immediately following test administration. School principals will receive the *Marking Guides* for the Senior 4 Pre-Calculus Mathematics (40S) Standards Test by January 20, 2004. The *Marking Guides* are to be stored in a secure location until 12:30 p.m. on January 20, 2004.

Students are expected to write the standards test as scheduled near the end of the semester in which they take the course (e.g., January for first-semester courses, June for second-semester and full-year courses).

All schools should receive the test materials by January 12, 2004. Principals of public schools receive test materials from their division/district offices. Principals of independent and band-operated schools receive test materials directly from the Assessment and Evaluation Branch.

The number of test booklets sent to each school is based on the October 3, 2003 registration information provided by the school. Please advise Carol Chabbert, Administrative Officer, at 945-0747 or toll-free at 1 800 282-8069, extension 0747, immediately if the number of students registered to write the test has increased since registration.

Manitoba Education, Citizenship and Youth supports local marking by:

- training representatives from local jurisdictions to act as local marking co-ordinators;
- collecting and scanning all *Answer/Scoring Sheets* from jurisdictions to provide them with test result summaries;
- centrally marking a random sample of locally-marked test booklets to provide feedback to jurisdictions on their local marking; and
- providing provincially-funded local jurisdictions with $10.00 for each test administered in accordance with the test schedule noted on the following page and marked locally. The calculation is based on the number of students whose completed marking scan sheets have been received.

Features of the test

The Senior 4 Pre-Calculus Mathematics (40S) Standards Test consists of two parts:

- Part 1—open-response questions where a scientific or graphing calculator may be used; and
- Part 2—selected-response, restricted-response and open-response questions where a calculator cannot be used.

Test schedule

The following table summarizes the schedule for the Senior 4 Pre-Calculus Mathematics (40S) Standards Test:

Date	Session	Preparation Time	Activity	Time Required for Test
Tuesday, January 20, 2004	1	15 minutes preparation and instructions	Part 1	60 minutes
			Part 2	120 minutes

The test must be written between 8:30 a.m. and 12:30 p.m. on Tuesday, January 20, 2004.

Policies for Standards Tests

Principals and supervising teachers should be aware that the *Policies and Procedures for Standards Tests* handbook has been distributed to all schools and is also available on our website at: http://www.edu.gov.mb.ca/ks4/assess/publications.html. This handbook outlines current policies and procedures related to the administration of standards tests and reflects the principles and guidelines contained in *Principles for Fair Student Assessment Practices for Education in Canada (1993).*

Adaptations

Schools are asked to identify and submit requests for test adaptations **at the time of student registration**. These requests should be submitted on the *Request for Adaptations Form* included with the registration package. Following departmental review of the adaptation requests received, schools will be advised of the department's decision regarding these requests (see *Policies and Procedures for Standards Tests*, Sections 4.0, 4.1 and 4.2).

Exemptions

Decisions for student exemptions that fall within the guidelines set out in *Policies and Procedures for Standards Tests* (Sections 5.0, 5.1 and 5.2) are made at the school level and do not require departmental approval. Schools are required, however, to report school-based exemptions on the *Student Exemption Report* included with the registration package. If additional student exemptions are needed after the registration deadline date, an additional *Student Exemption Report* should be submitted as soon as possible.

For exemptions that fall outside the parameters of policy and procedure guidelines, requests should be made **at the time of student registration**. Please submit requests in writing to Wenda Dickens, Acting Director, Assessment and Evaluation Branch, 71–1567 Dublin Avenue, Winnipeg, Manitoba R3E 3J5.

Re-read provisions

As the reporting of individual student scores on locally marked standards tests is the responsibility of each local jurisdiction, the re-read policy for these tests is determined locally. Should a re-read be required for a test booklet that has been sent to the department as a part of the feedback on the marking process, the department will return a photocopy of the test booklet upon request. Please contact Carol Chabbert, Administrative Officer, at 945-0747 or toll-free at 1 800 282-8069, extension 0747.

Return of tests to students

Local jurisdictions decide whether to keep the test booklets or return them to students. The department will return the random sample of test booklets to schools within three months of marking.

Test confidentiality

Test materials must be kept secure and sealed until administration.

Test booklets must not be reproduced nor information from them communicated in any way until test administration is completed.

The *Marking Guides* are to be stored in a secure location until the test administration is complete.

Student materials

Students may use the following materials as specified, during the Senior 4 Pre-Calculus Mathematics (40S) Standards Test:

- HB pencil;
- calculator (scientific or graphing—Part 1 only);
- eraser; and
- ruler.

Students are not permitted to exchange materials during the test.

The following materials must **not** be used during the test:

- class notes;
- notebooks;
- textbooks;
- dictionaries; and
- any electronic devices (phones, pagers, computers, etc.).

Students leaving the room

Students are expected to remain in the room for Part 1 (60 minutes) and until they have completed Part 2 of the test.

However, any student who must leave and re-enter the room should be accompanied by a replacement supervisor. This practice ensures that the supervising teacher can remain in the test room with the students at all times.

Late arrivals

Students who arrive more than one hour after the test has started are not allowed to write the test. Students who arrive late but within the first hour of the test are permitted to write but no extra time is allowed. The exception will be if students arrive late for reasons beyond their control. Under these circumstances, the local jurisdiction may allow extra time to write the test. The test however, may not start after 9:30 a.m. Please note that late students who are granted extra time

may only use their calculators for Part 1 of the test (maximum of 60 minutes). The classroom teacher should record on the *Supervising Teacher Report* the booklet numbers of students who arrived late and provide details of the circumstances which caused the student(s) to arrive late.

Student absences

Standards tests cannot be written at a later date and reported as a standards test result. Therefore, schools have the responsibility to provide a final mark to absent students. The final mark for these students must be identified as a school-based assessment and evaluation practice only (see *Policies and Procedures for Standards Tests*, Sections 6.0, 6.1 and 6.2 for additional information).

Students who formally refuse to write a standards test (by their own or by parental choice) or who are absent without a legitimate reason will receive a mark of 0% on the test.

Responding to student questions

Avoid making any directing/guiding comments during the test, other than those required for distribution of materials and instructions to students.

No explanations, translations, rephrasing, clarifying statements or comments of a similar nature may be provided about the test content unless otherwise specified. Supervising teachers can, however, provide clarification regarding test taking rules or procedures.

If a student asks a question related to test content, a suggested response is, "I'm sorry. I can't answer your question. Do your best with it."

Suspected errors in printing or wording should not be communicated to students unless specific instructions are received from the Assessment and Evaluation Branch. You may however contact Chantal Bérard, Assessment Consultant, at 945-7590 or toll-free at 1 800 282-8069, extension 7590 to report any suspected errors.

Violation of rules

The *Test Taking Rules* are read aloud to students by the supervising teacher immediately before the test (see Appendix 1, page 17).

Document any violation of rules on the *Supervising Teacher Report*. This documentation should include:

- the test booklet number; and
- a description of the circumstances, including available evidence.

The student may be permitted to complete the test.

The consequences of any violation of rules is a local decision.

Damaged or misprinted materials

Students should be assured that they will not be penalized for any obvious error in printing or wording. Local marking co-ordinators will be advised of any known errors before the marking session. Return the unused damaged or misprinted booklet(s) with the reports and forms being sent to the department immediately after the test session.

On test administration date

If a damaged or misprinted test booklet is discovered **at the start** of the test session:

- replace the damaged or misprinted test booklet with a new test booklet; and
- print "DAMAGED" on the test booklet cover and on the *Attendance Form*.

During test administration

If a damaged or misprinted test booklet is discovered **during** the writing of the test:

- give the student a new test booklet (the student keeps the damaged or misprinted test booklet until the end of the session);
- instruct the student to continue responding to test items in the new test booklet;
- collect both test booklets at the end of the test session and ensure that both booklets are kept together by bundling them with an elastic band;
- cross out the existing booklet number on the new test booklet and change it to the original booklet number so that **all student responses are recorded under the original booklet number**;
- indicate on the *Attendance Form* that two booklets have been used for one student (see Appendix 4, page 20); and
- record the damaged or misprinted test booklet number on the *Supervising Teacher Report*.

Responsibilities of School Principals

It is the responsibility of the principals to ensure that the tests are kept confidential until test administration is completed.

Other responsibilities are listed below.

If tests are not in your school by January 12, 2004:

- public school principals should contact their superintendent; and
- principals of independent and band-operated schools should contact Carol Chabbert, Administrative Officer, at 945-0747 or toll-free at 1 800 282-8069, extension 0747.

Use the *Packing List* to confirm that all materials have been received. Count the shrink-wrapped test booklets to ensure that there are enough booklets for the number of students writing the test. Keep the shrink-wrapped test packages unopened until the morning of the test. If additional test materials are needed, immediately contact Carol Chabbert, Administrative Officer.

Store test materials in a secure area in the school.

Provide supervising teachers with a listing of MET numbers for the students registered to write the test. Call the Professional Certification and Student Records Unit at 1 800 667-2378 to obtain MET numbers for new students.

Make arrangements to meet the requirements of the adaptations approved for your students (see Adaptations, page 3 and *Policies and Procedures for Standards Tests*, Sections 4.0, 4.1 and 4.2).

Co-ordinate test scheduling and space requirements.

Minimize the frequency of school announcements during the test session.

Ensure that a replacement supervisor is available to accompany students who must temporarily leave the room.

Provide the test materials to the supervising teacher on the morning of the test.

Complete the *Principal Feedback Form* before the end of the test session.

Advise the Assessment and Evaluation Branch of school closures or suspended school bus transportation services that occur on the test date as a result of unusual or extenuating circumstances or inclement weather conditions. Please contact Chantal Bérard, Assessment Consultant, at 945-7590 or toll-free at 1 800 282-8069, extension 7590.

Responsibilities of Supervising Teachers

Supervising teachers administer the test using the step-by-step procedures outlined in the following section.

(✓)	**Advance preparations**
	1. Check with your principal to ensure that:
	– appropriate scheduling and allocation of space for the test has occurred;
	– the test materials itemized on the *Packing List* have been inventoried;
	– a replacement supervisor is available to accompany students who must temporarily leave the room during the test;
	– arrangements have been made to accommodate the adaptations approved for students; and
	– a list of MET numbers for students writing the test is available.
	2. Ensure that all subject-related materials have been removed from the walls and boards.
	3. Notify students in advance of the:
	– time, location and duration of the test session;
	– materials needed to write the test;
	– *Test Taking Rules* (see Appendix 1, page 17);
	– minimum time a student must remain in the test room (60 minutes for Part 1 and until completion of Part 2).
	4. Ensure that you have extra supplies and that there is a functioning pencil sharpener and stapler in the room. Students must not exchange materials during the test.
	5. If there is more than one teacher supervising the test, photocopy the *Supervising Teacher Report*.
	6. Review the sample forms in the Appendices.
	7. Please note that the pre-printed *Demographic Data Forms* already have the students' names and MET numbers and should be distributed accordingly.
(✓)	**Prior to test session**
	1. Ensure that you have access to the following test materials:
	– *Packing List*;
	– *Part 1* and *Part 2* test booklets;
	– *Attendance Forms* (enclosed with shrink-wrapped test booklets);
	– *Student Registration Update Form*;
	– extra copies of test booklets (four extras);

	– *Demographic Data Forms* (pre-printed and generic);
	– a list of MET numbers for students writing the test;
	– *Answer/Scoring Sheets*;
	– *Supervising Teacher Report*;
	– *Supervising Teacher Feedback Form*;
	– *Principal Feedback Form* (this must be given to the principal and collected before returning materials);
	– *Administration Manual for Supervising Teachers* (this document); and
	– labelled envelope for return of materials to the Assessment and Evaluation Branch.
	2. Count the number of *Part 1* and *Part 2* test booklets. The number of test booklets should equal the number requested by the school plus the additional 4 booklets. If you need to split a bundle of booklets between classrooms, make a photocopy of the *Attendance Form*. Adjust each copy of the *Attendance Form* to reflect the booklets being used in each classroom.
	3. Arrange the desks so they are spaced as far apart as possible.
	4. Remove the following from the envelope labelled "For the Return of Demographic Data Forms":
	– pre-printed *Demographic Data Forms* in alphabetical order for every student registered to write the test. Ensure that the school name, school code, and MET numbers appearing on the pre-printed forms are accurate (see Appendix 2, page 18); and
	– generic *Demographic Data Forms* for students who joined the classroom after registration, and who are scheduled to write the test (see Appendix 3, page 19). **Note:** Only **one** *Demographic Data Form* should be completed for each student. If a pre-printed *Demographic Data Form* exists for a student, that is the form that should be used. **DO NOT** complete a generic *Demographic Data Form* for that student.
	5. Sort the pre-printed *Demographic Data Forms* for distribution.
	6. Make arrangements per school policy to ensure that students who are exempted from writing the test are accommodated in an alternate setting.
(✓)	**Beginning the test session**
	1. Distribute:
	– *Part 1* test booklets. *Part 2* test booklets are distributed after 60 minutes; and
	– *Demographic Data Forms*.
	2. Tell students that while they are working you will be circulating the *Attendance Form* so they can print their names beside their test booklet number.

	3. Read aloud the instructions for completing *Demographic Data Forms*. The instructions are found on the form itself. Direct students to print and shade in all the information requested. Remind students to accurately record the program (English, French Immersion, or Français) in which they are enrolled.
	4. Collect the *Demographic Data Forms*.
	5. Read aloud the *Test Taking Rules* to students (Appendix 1, page 17).
	6. Read aloud the *General Directions* to students (inside the front cover of the *Part 1* test booklet).
	7. Remind students to:
	– pace themselves in writing the test;
	– use the *Answer/Scoring Sheet,* which will be distributed with the *Part 2* test booklet, for responding to selected-response questions; and
	– check over their work to ensure that all questions have been attempted.
	8. **Instruct students to turn the page and begin. Record the start time for Part 1 on the *Supervising Teacher Report*.**
(✓)	**During the test session**
	1. Supervise the completion of the *Attendance Form* by ensuring that students print their name beside their test booklet number (see sample *Attendance Form*, Appendix 4, page 20). **Note:** This is the only record connecting students' identities to their *Part 1* and *Part 2* test booklets.
	2. To prepare for the distribution of Part 2 of the test, match-up each *Part 2* test booklet with the appropriate numbered *Answer/Scoring Sheet*. You may wish to insert the *Answer/Scoring Sheets* inside the front cover of the appropriate test booklet.
	3. Verify that all the information on each *Demographic Data Form* is accurately printed and shaded. Place all used forms in numerical order followed by any unused forms in the labelled envelope provided. Do not use elastic bands, staples or paper clips on these forms as they may damage the sheets. Do not seal the envelope.
	4. Half-way through Part 1 of the test session (after 30 minutes), advise students of the amount of time remaining for Part 1 by writing it on the board.
	5. After 60 minutes, ask the students to place their calculators on the floor for the remainder of the test session. Students retain their *Part 1* test booklet for the entire test session. Advise students they may continue working on Part 1 but they may not use their calculators once Part 2 is distributed.
	6. Distribute the *Part 2* test booklet and the *Answer/Scoring Sheet*, but **before** doing so please note: **The booklet numbers for *Part 1* and *Part 2* and the number on the *Answer/Scoring Sheet* need to be identical for each student. Advise students that as they receive the *Part 2* test booklet and an *Answer/Scoring Sheet* to verify that these items have the same number as their *Part 1* test booklet. Tell students that their answers to the selected-response questions are to be shaded on the back of the *Answer/Scoring Sheet*.**

	7. **Instruct students to turn the page and begin. Record the start time for Part 2 on the *Supervising Teacher Report*.**
	8. Half-way through Part 2 of the test session, advise students of the amount of time remaining by writing it on the board.
(✓)	**Ending the test session**
	1. Have the students place their *Answer/Scoring Sheets* inside the front cover of the *Part 1* test booklet. Collect both test booklets from each student.
	2. As students submit their test materials, ensure that each test booklet number matches the information on the *Answer/Scoring Sheet* and the *Attendance Form* (e.g., cross-reference student name and test booklet number). Each student must have the same booklet number on all test materials.
	3. Place a check "✓" under Part 1 and Part 2 of the *Attendance Form* in the "Booklets Submitted to LMC for Marking" column (see Appendix 4, page 20).
	4. **Record the time students finish writing the test on the *Supervising Teacher Report*.** Do not include extra time given for late arrivals or adaptations.
	5. Stack the test booklets in numerical order in 1 pile.
	6. If marking is conducted in the classroom or at the school, leave the *Answer/Scoring Sheets* inside the front cover of each *Part 1* test booklet. After marking, place all used *Answer/Scoring Sheets* in numerical order followed by any unused forms. Place the forms in the labelled envelope. Do not seal the envelope. If marking is conducted in a central location, place all used *Answer/Scoring Sheets* in numerical order followed by any unused forms. Place the forms in the labelled envelope. Do not seal the envelope.
	7. Complete the *Supervising Teacher Report*. Note any unusual circumstances on this report.
	8. Record and shade in the appropriate Reason Code on the pre-printed *Demographic Data Form* for all students who were absent for the session (see Appendix 2, page 18).
	9. Record and shade in the appropriate Drop-in Code on the generic *Demographic Data Form* for those students who were not included at the time of registration but were present to write the test (see Appendix 3, page 19).
	10. Ensure that the number of check marks in the "Booklets Submitted to LMC for Marking" column on the *Attendance Forms* matches your count of the booklets and *Answer/Scoring Sheets* for submission.
	11. Complete and check the *Student Registration Update Form*. Use the reason codes (Appendix 5, page 21) provided.
	12. Make two copies of the *Attendance Forms* and *Supervising Teacher Report*. One copy must be retained by the school and one forwarded to the local marking site, if a centralized marking model is used.
	13. Collect the *Principal Feedback Form*.

	14. Complete the *Supervising Teacher Feedback Form.*
	15. Return all test booklets and other test-related materials to the secure area in the school until commencement of marking or until materials are to be forwarded to the local marking site.
	16. Send materials according to the instructions on the following pages.

Points to Remember

- Remain in the classroom throughout the entire test session and engage in work solely related to the duties of supervising the test.
- A replacement supervisor must be available to assist you.
- Store all documents not in use in a secure area in the school.
- Ensure that once a booklet number has been assigned to a student, all materials distributed to that student have the same booklet number.
- Students **may** review or complete work from Part 1 during the Part 2 portion of the test session. Students **cannot**, however, use their calculator during Part 2 of the session.
- Students **may not** bring anything produced outside of the test room.

Submitting materials on January 20, 2004 (the end of the test)

It is essential that the following materials are packaged and forwarded to the local marking site immediately after completion of the test if a centralized model is used.

- **All used** test booklets;
- **All used** *Answer/Scoring Sheets*;
- Copy of the *Attendance Forms*; and
- Copy of the *Supervising Teacher Report*.

All other documentation should be verified and returned to the Assessment and Evaluation Branch after the test session.

- *Attendance Forms*;
- **All used** *Demographic Data Forms* (in envelope provided);
- *Student Registration Update Form*;
- *Supervising Teacher Report*;
- *Supervising Teacher Feedback Form*;
- *Principal Feedback Form*; and
- **All** damaged or misprinted booklets.

Schools in the City of Winnipeg return their envelopes to the Assessment and Evaluation Branch by calling Mid-Canada Courier (989-5630) and arranging pick-up between **1:00 p.m. and 4:00 p.m.** on **January 20, 2004**. The person placing the call should quote **account number E901**, provide the school name, address and the name of the contact person.

Schools outside the City of Winnipeg return their envelopes to the Assessment and Evaluation Branch by using the Xpresspost Flexipack envelope provided and taking it to the post office.

Submitting materials after local marking

(to be received by the Assessment and Evaluation Branch no later than January 30, 2004)

When local marking is done at a central location, the **local marking co-ordinator** is responsible for sending the materials to the Assessment and Evaluation Branch. When local marking is done at the classroom level, the **teacher** is responsible for sending the materials to the Assessment and Evaluation Branch.

After local marking has been completed, prepare a package to be received by the Assessment and Evaluation Branch no later than January 30, 2004. This will ensure there is sufficient time for processing.

The package must include:

- **All** *Answer/Scoring Sheets*;
- Test booklets requested by the department for feedback on the marking process;
- *Sample Request List*; and
- *Local Marking Co-ordinator Feedback Form.*

Jurisdictions within the City of Winnipeg send packages directly to the Assessment and Evaluation Branch via **Mid-Canada Courier (989-5630)**. The person placing the call should quote **account number E901**, give the jurisdiction name, address, and the name of the contact person.

Jurisdictions outside the City of Winnipeg return their package by Xpresspost. It is important to package all materials in the **Xpresspost Flexipack envelope** provided whenever possible.

After the package is sealed, deliver it to the local post office.

We have provided sufficient materials for the return of **one** envelope/package. If you must send a second envelope/package, photocopy the mailing address label and affix the copy to the second package. Note you must use a photocopy of the mailing address label as it provides Canada Post with billing information. Ask the postal clerk for an additional bar code label to attach to the second envelope/package. Ensure that the school representative retains the bar code number in case tracing the package is required.

Return **all** *Answer/Scoring Sheets*. We remind you that payments will be made according to the number of completed marking sheets received.

Appendices

Appendix 1: Test Taking Rules

Senior 4 Pre-Calculus Mathematics (40S) Standards Test

The supervising teacher is expected to read these rules aloud to students at the beginning of the Senior 4 Pre-Calculus Mathematics (40S) Standards Test:

- Check the title of your test booklets carefully.
- During this test session, do not proceed until instructed to do so.
- If you receive a damaged or misprinted booklet, raise your hand and I will give you a new one.
- You are expected to remain in the test room for the allotted time for Part 1 and until you complete Part 2. (You may leave before that time in exceptional circumstances such as illness, when accompanied by a supervisor.)
- At the beginning of the session you should only have a *Part 1* test booklet, an HB pencil, an eraser, a ruler and your calculator (scientific or graphing, for Part 1 only). No other materials are permitted. Do not exchange test materials. When the *Part 2* test booklet is distributed, you will also receive an *Answer/Scoring Sheet* to record your answers to the selected-response questions. When I hand out the *Part 2* test booklet make sure the numbers on the *Part 1* and *Part 2* test booklets match the number on your *Answer/Scoring Sheet*.
- Your calculator must remain on the floor throughout the administration of Part 2. You may keep your *Part 1* test booklet for the entire session and continue working on Part 1 if you wish, **but** you may not use your calculator.
- You will not give assistance to, nor receive assistance from another student. If you require something during the test, raise your hand and I will come to you.
- During the test I can help you with the directions, but I will not be able to help you with the test questions.
- Do **not** identify yourself, your school, teacher or town in your test booklet.
- When I ask you to stop writing, put your pencil down immediately.
- You may not discard any materials. The test booklet must remain intact unless otherwise instructed. Any extra paper used must be stapled into the test booklet on the appropriate page. You may **not** leave the room with any materials pertaining to the test.

Appendix 2: Sample Pre-printed Demographic Data Form

DEMOGRAPHIC DATA FORM / FORMULAIRE DE DONNÉES DÉMOGRAPHIQUES

SENIOR 4 PRE-CALCULUS MATHEMATICS STANDARDS TEST – JANUARY 2004

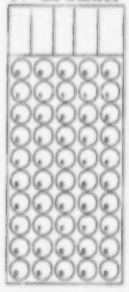
TEST BASÉ SUR LES NORMES MATHÉMATIQUES PRÉ-CALCUL DE SECONDAIRE 4 – JANVIER 2004 1

School/ École	**9999 SCHOOLNAME**	Program/ Programme	**ENGLISH**
Name/ Nom	**SURNAME, GIVEN NAME**		
Birth Date/ Date de naissance	**01/01/01**	MET #/ N° MET	**999-999-999**

INSTRUCTIONS TO THE STUDENT:

1. Write and shade in the **Booklet No.**
2. Use an HB pencil only. Do not use ink, ball point, or felt tip pens.
3. Erase cleanly any marks you wish to change.
4. Do not mark other areas on this form.
5. If the pre-printed information is inaccurate, please cross it out and write in the correct information.

Booklet No./ N° de cahier



DIRECTIVES À L'ÉLÈVE :

1. Inscrire votre **N° de cahier** et noircir les cercles qui y correspondent.
2. Utiliser seulement un crayon HB. Ne pas utiliser de stylo à encre ou à bille, ni de crayon feutre.
3. Bien effacer toute marque que vous désirez changer.
4. Ne pas faire de marques à d'autres endroits sur ce formulaire.
5. Si les renseignements pré-imprimés sont inexacts, les rayer et écrire les renseignements exacts.

FOR TEACHER USE ONLY:

Write and shade in the **Reason Code** if the student is absent for **all** test sessions. Please see list of reason codes and instructions enclosed.

Reason Code/ Code désigne

Other/Autre :

À L'USAGE DE L'ENSEIGNANT SEULEMENT :

Écrire le Code désigné et noircir les cercles qui y correspondent si l'élève est absent pour **toutes** les séances de test. Veuillez vous reporter à la liste de codes et aux directives ci-incluses.

0000

Appendix 3: Sample Generic Demographic Data Form

DEMOGRAPHIC DATA FORM

Manitoba

NAME (Last)

NAME (First)

INSTRUCTIONS

- Use an HB pencil only.
- Do not use ink, ballpoint, or felt tip pens.
- Shade the circle completely.
- Erase cleanly any marks you wish to change.
- Do not mark other areas on this form.

EXAMPLES

INCORRECT	CORRECT
Ⓐ Ⓑ Ⓒ Ⓓ	Ⓐ Ⓑ Ⓒ ●
Ⓐ Ⓑ Ⓒ Ⓓ	
Ⓐ Ⓑ Ⓒ Ⓓ	
Ⓐ Ⓑ Ⓒ Ⓓ	

PROGRAM INFORMATION

- Ⓔ English Program
- Ⓘ French Immersion Program
- Ⓕ Français Program

PROVINCIAL STUDENT (MET) NO.

BIRTH DATE

MONTH	DAY	YEAR
Jan		
Feb		
Mar		
Apr		
May		
June		
July		
Aug		
Sept		
Oct		
Nov		
Dec		

BOOKLET NO.

SCHOOL CODE

REASON CODE

NOTE TO TEACHER:
Complete the Reason Code only for students who "dropped in" to the school and are present to write the test.

Appendix 4: Sample Attendance Form

School Number: 9999
Somewhere School
Somewhere Address
Somewhere MB R0R 0R0

Page _____ of _____

ATTENDANCE FORM—JANUARY 2004

SENIOR 4 PRE-CALCULUS MATHEMATICS (40S) STANDARDS TEST

Booklet Number	Student Name (*please print*)	Booklets Submitted to LMC* for Marking (*check* ✓)	
		Part 1	Part 2
30001	Karen Masi	✓	went home sick
30002	Walter Woronowski	✓	✓
30003	Michael Smith	✓	✓
30004	DAMAGED		
30005	Jason Chu	✓	✓
30006	Shelley Bracken (also 30008)	✓	✓
30007	Bonnie Walters	✓	✓
30008	Used with 30006 (damaged)		
30009	UNUSED		
30010	UNUSED		

Note: Record any absences on the *Student Registration Update Form*.

Mr. John Ormiston
Supervising Teacher's Name (*please print*)

Please retain a copy of this form for your records. You will need a copy of this form to link your locally marked test with a student's name.

***Local marking co-ordinator**

Appendix 5: List of Reason Codes to Record Changes in Standards Test Student Registration Numbers

(Please use these codes to complete the *Student Registration Update Form* and the *Demographic Data Form* before returning them to the Assessment and Evaluation Branch in January 2004.)

Absent	
01	arrived too late to write the test
02	did not write due to weather
03	family/personal reasons
04	medical/illness/accident
05	out-of-town activity (athletic/academic)
06	religious objections
07	student refused to write
08	unexcused
09	working
19	other (please specify)

Withdrawn	
22	dropped course
23	incarcerated/court
24	medical/illness/accident
25	family/personal reasons
26	poor attendance/progress
27	quit school
29	other (please specify)

Incomplete	
40	continuing course/program next semester
41	no work completed
49	other (please specify)

Errors in Registration	
20	changed program
21	deceased
50	already has credit
51	auditing the course
52	exchange student
53	registered twice
54	quarter system projected registration
55	registered in wrong grade
59	other (please specify)

Exemptions	
60	emotional/psychological condition or circumstances
61	"M" credit designation
62	"E" credit designation
63	"phase d'accueil—bloc intensif" designation
69	other (please specify)

Drop Ins	
70	forgotten on original test registration
71	late registration to course
72	transferred in from another school/province/country
73	rewrite (not currently enrolled)
79	other (please specify)

Migrancy	
90	transferred to another school within the province
91	transferred to another school outside of the province
99	other (please specify)

Senior 4 Pre-Calculus Mathematics (40S) Standards Test

Part 1

January 2004

Manitoba

Building for the Future

Pre-Calculus Mathematics (40S)
Standards Test

DESCRIPTION

Total Possible Marks: 100 **Time: 3 hours**

Part	Description	Time	Value
Part 1	• **Open Response:** 10 questions worth a total of 35 marks Calculators (scientific or graphing) are permitted.	60 minutes	35%
Part 2	• **Selected Response:** 15 questions worth 1 mark each • **Restricted Response:** 15 questions worth 1 mark each • **Open Response:** 11 questions worth a total of 35 marks Calculators are not permitted.	120 minutes	65%

GENERAL DIRECTIONS

- Formulas are given on the tear-out "Formula Sheet" at the beginning of this test booklet. Please do not tear out any pages other than the "Formula Sheet".
- Read all instructions carefully.
- The blank pages at the back of each booklet may be used for rough work. No marks will be given for work done on these pages.
- Note that diagrams and graphs provided in the test booklets may not be drawn to scale.
- When Part 2 is distributed, you may keep Part 1 in case you wish to continue working on Part 1 questions (without a calculator).

Pre-Calculus Mathematics (40S) Formula Sheet

$$S = r\theta$$

$$\sin^2\theta + \cos^2\theta = 1$$

$$\tan^2\theta + 1 = \sec^2\theta$$

$$1 + \cot^2\theta = \csc^2\theta$$

$$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$$

$$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$$

$$\tan(\alpha - \beta) = \frac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$$

$$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$$

$$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$$

$$\tan(\alpha + \beta) = \frac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$$

$$\sin 2\alpha = 2\sin\alpha\cos\alpha$$

$$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha = 1 - 2\sin^2\alpha = 2\cos^2\alpha - 1$$

$$\tan 2\alpha = \frac{2\tan\alpha}{1 - \tan^2\alpha}$$

$$P(n,r) \text{ or } {}_nP_r = \frac{n!}{(n-r)!}$$

$$C(n,r) \text{ or } {}_nC_r = \frac{n!}{r!(n-r)!}$$

$$t_{k+1} = {}_nC_k a^{n-k} b^k$$

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$A = Pe^{rt}$$

$$e \approx 2.71828$$

$$\log_a(MN) = \log_a M + \log_a N$$

$$\log_a\left(\frac{M}{N}\right) = \log_a M - \log_a N$$

$$\log_a(M^n) = n\log_a M$$

$$\log_a M = \frac{\log_b M}{\log_b a}$$

$$(x-h)^2 + (y-k)^2 = r^2$$

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1, \quad a > b$$

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1, \quad a > b$$

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

$$(y-k) = a(x-h)^2$$

$$(x-h) = a(y-k)^2$$

$$t_n = t_1 r^{n-1}$$

$$S_n = \frac{t_1(1-r^n)}{1-r} = \frac{t_1(r^n-1)}{r-1}$$

$$S_n = \frac{t_1 - t_n r}{1-r}$$

$$S_\infty = \frac{t_1}{1-r}, \quad |r| < 1$$

Open-Response Questions

Instructions

- There are 10 questions for a total of 35 marks.
- Calculators (scientific or graphing) are allowed for this part of the test.
- Write each solution in the space provided.
- For full marks, your answers must show all pertinent diagrams, calculations, and explanations.
- Graphing calculator solutions should include an explanation of how your final answer is obtained.
- Your solutions should be neat, organized and clear.
- If any curve contains asymptotes, the asymptotes should be included in the graph.

DO NOT USE

Open-Response Questions

1. An investment earns interest at an annual rate of 7% compounded semi-annually. How long will it take, in years, for the investment to triple? 101

(3 marks)

2. Find and simplify the middle term of $(2a-b)^8$. 102

(3 marks)

DO NOT USE

3. Solve the equation $3^{x+1} = 4(5^x)$ **algebraically**. 103
Express your final answer correct to 3 decimal places.

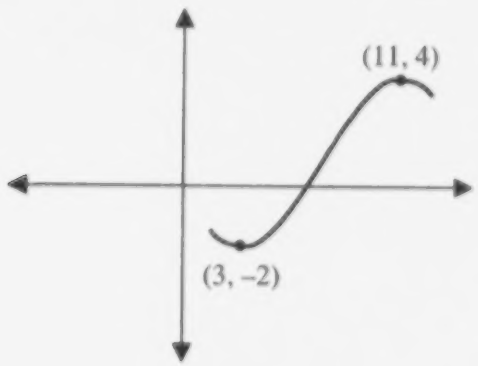
(4 marks)

4. Evaluate: $\sum_{k=1}^{\infty} 2^{6-k}$. 104

(3 marks)

DO NOT USE

5. A minimum and a maximum of a sinusoidal function are shown below.



a) Find an equation of this function. 105

(4 marks)

b) Find the y-intercept. State the answer correct to 3 decimal places. 106

(1 mark)

DO NOT USE

6. If $\cos\alpha = -\frac{3}{5}$ where $\tan\alpha > 0$ and $\sin\beta = \frac{5}{13}$ where $\sec\beta > 0$, find the exact value of $\sin(\alpha+\beta)$. 107

(4 marks)

DO NOT USE

7. In how many ways can 11 friends sit at two round tables if one table seats 6 and the other table seats 5?
Briefly explain your calculations.

108

(3 marks)

8. Ron is having a dinner party. He invites 7 out of his 10 friends.

a) In how many ways can he choose his 7 guests?

109

(1 mark)

b) In how many ways can he choose his guests if two of them, Al and Sue, must attend together or not at all?
Briefly explain your calculations.

110

(3 marks)

DO NOT USE

9. In a Manitoba high school, 30% of the students are in Senior 2, 25% of the students are in Senior 3 and the rest of the students are in Senior 4.
Francis ran for student president.
The results were:
40% of the Senior 2 students voted for her.
20% of the Senior 3 students voted for her.
60% of the Senior 4 students voted for her.

a) What is the probability that a randomly selected Senior 4 student did **not** vote for Francis? 111

(1 mark)

b) If a student voted for Francis, what is the probability that the student was in Senior 4? 112

(3 marks)

DO NOT USE

10. Write an equation of the circle which has the centre (2, –5) and is tangent to the x-axis. 113

(2 marks)

NO MARKS WILL BE AWARDED FOR WORK DONE ON THIS PAGE

NO MARKS WILL BE AWARDED FOR WORK DONE ON THIS PAGE

NO MARKS WILL BE AWARDED FOR WORK DONE ON THIS PAGE

NO MARKS WILL BE AWARDED FOR WORK DONE ON THIS PAGE

NO MARKS WILL BE AWARDED FOR WORK DONE ON THIS PAGE

Senior 4 Pre-Calculus Mathematics (40S) Standards Test

Part 2

January 2004

Manitoba
Building for the Future

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Manitoba Education, Citizenship and Youth.
Assessment and Evaluation Branch,
Winnipeg, Manitoba R3E 3J5

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Selected-Response Questions

Instructions

- There are 15 selected-response questions.
- Each question is worth 1 mark.
- Read each question carefully.
- Examine all the choices before you decide on your answer.
- You may use the spaces beside each question for rough work.
- Record your answers on the sheet provided.
- Provide only one answer per question.
- There is no penalty for guessing.
- Answer all questions.
- Calculators (scientific or graphing) are **not** allowed for this part of the test.

Selected-Response Questions

11. If $\sec\theta = \frac{5}{3}$, then the value(s) of $\tan\theta$ is (are):

a) $\frac{3}{4}$

b) $\frac{4}{3}$

c) $\pm\frac{3}{4}$

d) $\pm\frac{4}{3}$

12. If a wheel with a radius of $\frac{\pi}{2}$ cm travels a distance of 35π cm, then the wheel has rotated an angle of :

a) 70

b) $\frac{35\pi^2}{2}$

c) $\frac{35}{2}$

d) 70°

13. Which of the following statements is true for the function $f(x) = \sin x$?

a) $f(x) = f(-x)$

b) $f(x) = -f(x)$

c) $f(x) = -f(-x)$

d) $f(x) = f^{-1}(x)$

14. If x is an acute angle, which of the following statements is **not** true?

a) $0 < \cos x < 1$

b) $0 < \sin x < 1$

c) $\sin x \leq \tan x$

d) $\sin^2(x) + \cos^2(x) = 1$

15. An equivalent form of $\cos\left(\frac{\pi}{2} - x\right)$ is:

a) $\cos x$

b) $\sin x$

c) $-\cos x$

d) $-\sin x$

16. If $g(x) = 6\sin(2x)$ and $h(x) = \frac{1}{2}g(x)$, then the amplitude of $h(x)$ is:

a) 1

b) 3

c) 6

d) 12

17. The range of $y = 3\csc x$ is:

a) $(-\infty, -3] \cup [3, \infty)$

b) $(-\infty, \infty)$

c) $\left[-\frac{1}{3}, \frac{1}{3}\right]$

d) $[-3, 3]$

18. The range of $y = -2^x - 1$ is:

a) $y > 0$

b) $y < 0$

c) $y > -1$

d) $y < -1$

19. The equation $4\log 3 + x\log 3 = 3\log 4$ is equivalent to:

a) $(4 + x)^3 = 4^3$

b) $3^4 + 3^x = 4^3$

c) $3^{x+4} = 4^3$

d) $3^{4x} = 4^3$

20. If $f(x)=8\left(2^{-x}\right)$, then the value of $f(-1)$ is:

a) -4

b) $\frac{1}{16}$

c) 4

d) 16

21. In the expansion of $(x+y)^3$, if c equals the numerical coefficient of xy^2, and d equals the numerical coefficient of x^2y, then:

a) $c>d$

b) $c=d$

c) $c<d$

d) $c+d<0$

22. The term in the expansion of $\left(x^2+\frac{1}{x}\right)^6$ which does **not** contain an x would be:

a) t_4

b) t_5

c) t_6

d) t_7

23. Randomly selecting a day of the week and a month of the year, what is the probability of selecting Sunday and June?

a) $\frac{1}{84}$

b) $\frac{18}{84}$

c) $\frac{19}{84}$

d) $\frac{7}{12}$

24. A committee of 2 girls and 2 boys is randomly selected from 7 girls and 5 boys. How many different committees are possible?

a) $\frac{12!}{4!8!}$

b) $\frac{12!}{2!5!2!3!}$

c) $\frac{7!5!}{5!3!}$

d) $\frac{7!5!}{5!2!2!3!}$

25. The length of the major axis in the graph of $\frac{x^2}{5}+\frac{y^2}{9}=1$ is:

a) $\sqrt{5}$

b) $2\sqrt{5}$

c) 3

d) 6

Restricted-Response Questions

Instructions

- There are 15 restricted-response questions.
- Each question is worth 1 mark.
- Calculators are **not** allowed for this part of the test.
- Write your final answer in the space provided.
- Work need not be shown in order to obtain full marks for a correct answer. However, in the case of an incorrect final answer, part marks may be awarded only if work is shown.
- Solutions need not have rationalized denominators.

DO NOT USE

Restricted-Response Questions

26. Convert $\frac{7\pi}{18}$ to degrees. Simplify your answer. 114

(1 mark)

27. Give a possible value for $\sin\theta$ if $\cos\theta = \frac{1}{\sqrt{10}}$. 115

(1 mark)

28. Simplify $\cos^2(\alpha+\beta)+\sin^2(\alpha+\beta)$. 116

(1 mark)

DO NOT USE

29. What is the period of $g(x)=2\sin(3x)-1$? 117

(1 mark)

30. The coordinates of a point on the graph of $y=h(x)$ are (–3, 5). 118
State the coordinates of a point on the graph of $y=h(-x)$.

(1 mark)

31. If $\log_a 2=p$ and $\log_a 3=q$, find an expression for $\log_a 6$ in terms of p and q. 119

(1 mark)

DO NOT USE

32. Identify the conic represented by the equation: $x^2 - 4x = y^2 + 2$. 120

(1 mark)

33. A geometric sequence has $t_2 = 100$ and $t_4 = 4$.
Find a possible value of the common ratio, r. 121

(1 mark)

34. How many five-digit arrangements are possible using all of the digits 1, 2, 3, 4, 5, if the middle digit must be odd and repetition of digits is not allowed? 122

(1 mark)

DO NOT USE

35. The probability of hitting a target is $\frac{1}{3}$. Therefore, the probability of missing on 3 consecutive tries is $\frac{8}{27}$.

What is the probability of hitting this target at least once in 3 tries?

123

(1 mark)

36. State the coordinates of the vertex of the relation $y^2 = 4x + 4$.

(1 mark)

124

37. If $\sin(5\theta) = \frac{\sqrt{3}}{2}$, where 5θ is an acute angle, find the exact value of θ.

(1 mark)

125

DO NOT USE

38. Find $f^{-1}(x)$, if $f(x)=e^x$. 126

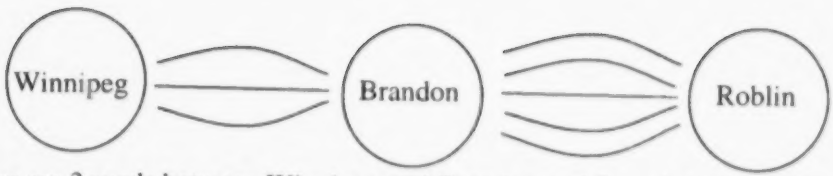
(1 mark)

$f^{-1}(x)=$_____

39. A box contains 2 red, 3 black and 2 white balls. A ball is selected at random from the box. What is the probability that the ball is **not** white? 127

(1 mark)

40. 128



There are 3 roads between Winnipeg and Brandon and 5 roads between Brandon and Roblin. How many different routes are there from Winnipeg to Roblin through Brandon?

(1 mark)

Open-Response Questions

Instructions

- There are 11 questions worth a total of 35 marks.
- Calculators are **not** allowed for this part of the test.
- Write each solution in the space provided.
- For full marks, your answers must show all pertinent diagrams, calculations, and explanations.
- Your solutions should be neat, clear and well organized.
- If any curve contains asymptotes, the asymptotes should be included in the graph.
- Solutions need not have rationalized denominators.

DO NOT USE

Open-Response Questions

41. Find the coordinates of the point of intersection of the unit circle with the line segment joining the origin to the point (–2, –1). 129

(3 marks)

42. Change the following equation to standard form: 130

$$5y^2 + 50y - 4x^2 + 24x + 69 = 0$$

(3 marks)

43. Solve for x:

$$\log_2\left(\log_{81} x\right) = -2$$

(3 marks)

DO NOT USE

44. Given the function $f(x)=\sqrt{x+4}-3$, find:

(4 marks)

a) x-intercept of $f(x)$ _____

132

b) domain of $f(x)$ _____.

133

c) x-intercept of $f^{-1}(x)$ _____

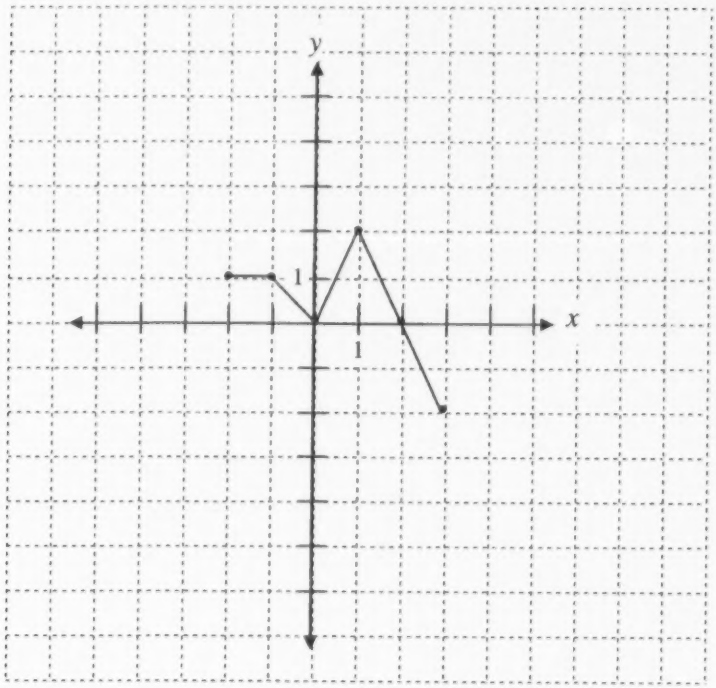
134

d) domain of $f^{-1}(x)$ _____

135

DO NOT USE

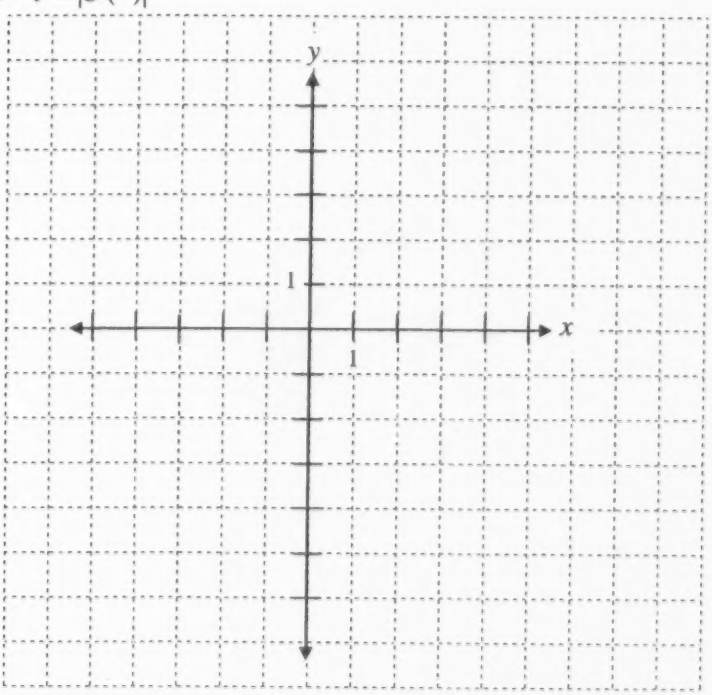
45. The graph of $y = g(x)$ is given below.



Sketch:

a) $y = |g(x)|$

(1 mark)



136

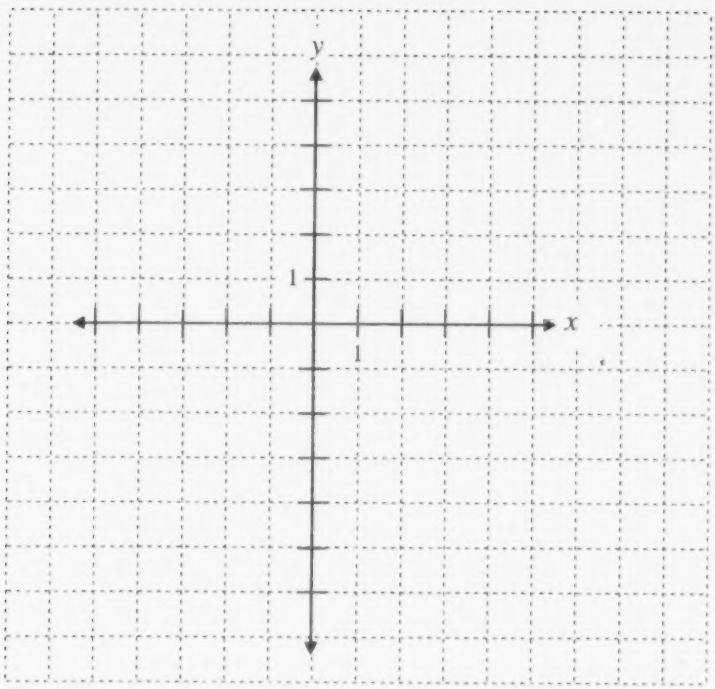
DO NOT USE

Sketch:

b) $y = -g(x+2)$

(2 marks)

137

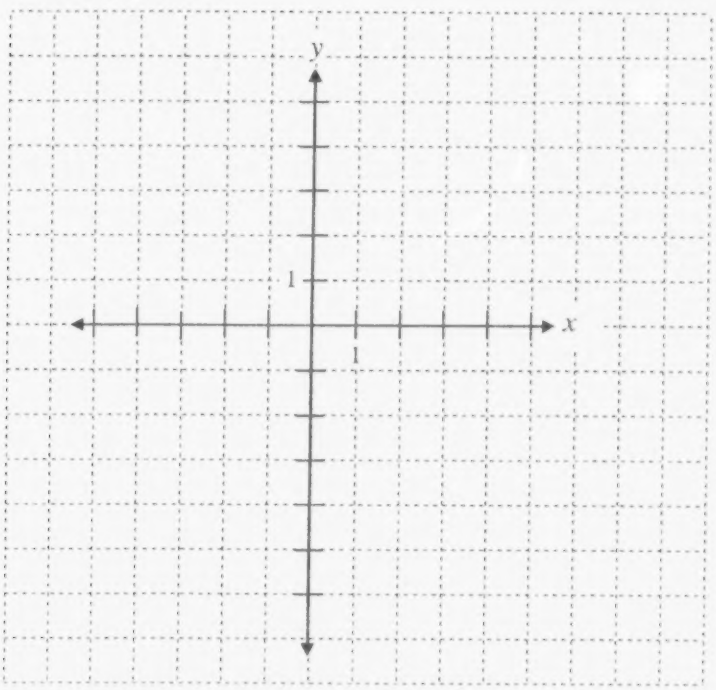


Sketch:

c) $y = 3g(x) - 1$

(2 marks)

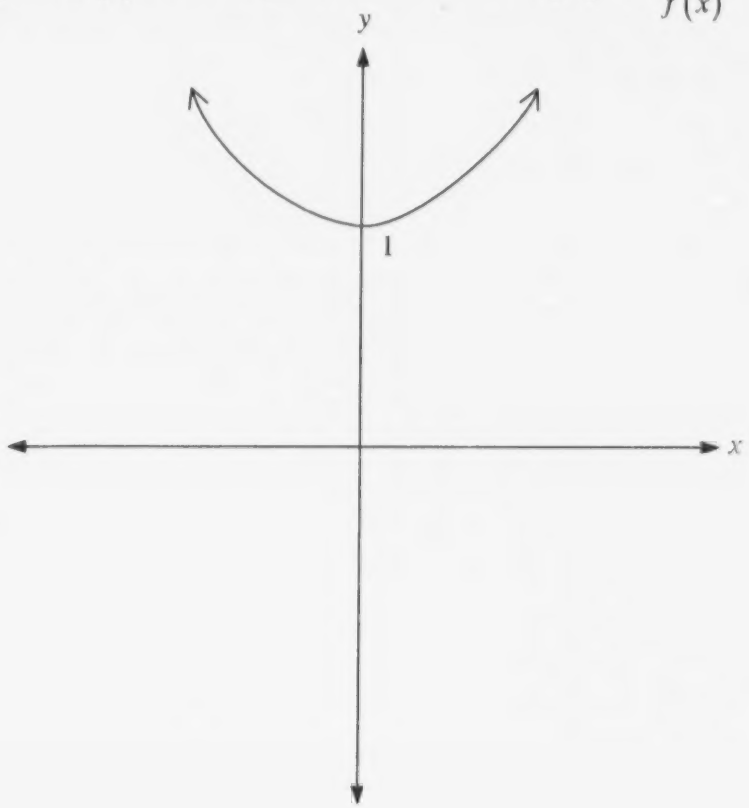
138



DO NOT USE

46. The graph of $f(x) = x^2 + 1$ is given below. Sketch the graph of $\frac{1}{f(x)}$.

(2 marks)



139

47. Find the exact value of $\tan(75°)$.

140

(3 marks)

DO NOT USE

141

48. Prove the following identity:

$$\frac{1-\tan^2\theta}{1+\tan^2\theta}=\cos 2\theta$$

(3 marks)

DO NOT USE

49. Solve for x:

142

$$\left(\frac{1}{4}\right)^{2x-1} = 8^{x}$$

(2 marks)

50. Solve for x, where $0 \leq x \leq 2\pi$:

143

$$\sec^2 x - \sec x = 2$$

(4 marks)

DO NOT USE

144

51. Jar A contains 1 yellow and 2 red marbles.
Jar B contains 2 yellow and 1 red marbles.
A marble is chosen at random from Jar B and placed in Jar A.
Find the probability of now choosing a red marble from Jar A.

(3 marks)

NO MARKS WILL BE AWARDED FOR WORK DONE ON THIS PAGE

NO MARKS WILL BE AWARDED FOR WORK DONE ON THIS PAGE

NO MARKS WILL BE AWARDED FOR WORK DONE ON THIS PAGE

NO MARKS WILL BE AWARDED FOR WORK DONE ON THIS PAGE

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Senior 4 Pre-Calculus Mathematics (40S) Standards Test

Marking Guide

January 2004

Manitoba
Building for the Future

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Manitoba Education, Citizenship and Youth,
Assessment and Evaluation Branch,
Winnipeg, Manitoba R3E 3J5

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Part 2: Open-Response Questions 45

General Marking Instructions

Please make no marks in the student test booklets. If the booklets have marks in them, the marks need to be removed by department staff prior to feedback marking should the booklet be selected.

Please ensure that:

- the student test booklet number and the number on the *Answer/Scoring Sheet* are identical;
- **only a pencil is used to complete the *Answer/Scoring Sheets*, by both students and markers;**
- the totals of each of the four parts are written at the bottom;
- each student's final result is recorded, by test booklet number, on the corresponding ***Answer/Scoring Sheet***;
- the grand total is written in the appropriate location;
- the *Answer/Scoring Sheet* is complete; and
- a photocopy has been made for school records.

Once marking is completed, please forward the ***Answer/Scoring Sheets*** to Manitoba Education, Citizenship and Youth in the envelope provided (for more information see the *Administration Manual for Supervising Teachers*).

Scoring the Selected-Response Questions

Please follow the instructions on the ***Answer Key for Selected-Response Questions*** on page 5.

Scoring the Restricted-Response and Open-Response Questions

The remainder of Senior 4 Pre-Calculus Mathematics (40S) Standards Test is composed of Restricted-Response questions and Open-Response questions. Restricted-Response questions are worth 1 mark each and Open-Response questions are worth 2 to 5 marks each.

Each question is designed to elicit a well-defined response according to the associated specific learning outcome(s) and relevant mathematical processes. Their purpose is to determine whether a student meets the standards for the course as they relate to the knowledge and skills associated with the question.

To receive full marks, a student's response must be complete and correct. Otherwise, 1 mark must be deducted for each concept error, and ½ mark must be deducted for a calculation or arithmetic error. Where alternative answering methods are possible, the *Marking Key* attempts to address the most common solutions.

Assistance

If, during marking, any marking issue arises that cannot be resolved locally, please call Manitoba Education, Citizenship and Youth at the earliest opportunity to advise and/or seek assistance. If required, we will connect you to an advisor. Also, please be sure to notify us before making any modifications to the answer keys or scoring rubrics. Contact person: Chantal Bérard at 945-7590 or toll-free at 1 800 282-8069, extension 7590.

SCORING GUIDELINES

Answer Key for Selected-Response Questions

Answer Key for Selected-Response Questions

If you are using this sheet by punching out holes corresponding to the correct answers and overlaying this sheet on the *Answer/Scoring Sheet*, then

1) check the student's responses for multiple bubbles first (questions with multiple bubbles are to be scored as "0");

2) overlay this page on the *Answer/Scoring Sheet*; and

3) count the number of correct responses, excluding questions with multiple responses, if any.

You may also score the multiple-choice questions by making a transparency of this page and overlaying it on the *Answer/Scoring Sheet*.

Remember to write the total score for the multiple-choice questions at the bottom of the *Answer/Scoring Sheet*.

SELECTED RESPONSE QUESTIONS / QUESTIONS À RÉPONSE CHOISIE

Fill in the best answer for each question. / Choisir la meilleure réponse pour chaque question.

11	D	16	B	21	B
12	A	17	A	22	B
13	C	18	D	23	A
14	C	19	C	24	D
15	B	20	D	25	D

Selected-Response Questions			
Question	*Outcome*	*Question*	*Outcome*
11	A2	19	D5
12	A1	20	D1
13	B3, A6	21	E4
14	A6, C1	22	E4
15	C2	23	G2
16	B7	24	E3
17	B7, A6	25	F3
18	D1		

SCORING GUIDELINES

Marking Key
including student exemplars

Part 1—Open-Response Questions

Question No. 1

Outcome: D8

Question

1. An investment earns interest at an annual rate of 7% compounded semi-annually. How long will it take, in years, for the investment to triple?

3 marks

Solution

$3x = x\left(1+\frac{0.07}{2}\right)^{2t}$ — 1 mark for substitution in formula

$3 = 1.035^{2t}$ — ½ mark for simplification

$\log 3 = 2t \log 1.035$ — ½ mark for log theorem

$t = \frac{\log 3}{2\log 1.035}$ — ½ mark for solving for t

$t = 15.968$ — ½ mark for consistent answer

It will take 15.968 or 16

Note(s)

- give full marks if x is omitted in the first line
- since "years" is specified in the question it doesn't need to be stated with the final answer
- precision of final answer is not specified therefore accept any correctly rounded or truncated answer
- if student rounds any calculation prior to the final answer, which results in an incorrect answer, deduct ½ mark for arithmetic error

Question No. 2

Outcome: E4

Question

2. Find and simplify the middle term of $(2a-b)^8$. 3 marks

Solution

$t_5 = {}_8C_4(2a)^4(-b)^4$ OR $\binom{8}{4}(2a)^4(-b)^4$ — 1½ marks for setup { ½ mark for ${}_8C_4$; ½ mark for $(2a)^4$; ½ mark for $(-b)^4$ }

$= 70(16a^4b^4)$ — 1 mark for simplification (½ mark for each factor)

$= 1120a^4b^4$ — ½ mark for final answer

Note(s)

- if student only writes t_5 give ½ mark
- if student finds term four, give maximum 2 marks
  $t_4 = {}_8C_3(2a)^5(-b)^3 = -1792a^5b^3$
- second step is optional

EXEMPLARS

2. Find and simplify the middle term of $(2a-b)^8$.

$(2a-b)^8$ let $a = x$ and $b = y$

middle term = 4th term.

$8C3\, x^5 y^3 = 56(2a)^5(-b)^3$

$= 56\, 2a^5 - b^3$

Mark:	½ out of 3
Note(s):	• gave ½ mark for 56

Question No. 3

Outcome: D6

Question

3. Solve the equation $3^{x+1} = 4(5^x)$ **algebraically**.
   Express your final answer correct to 3 decimal places.

4 marks

Solution

$(x+1)\log 3 = \log 4 + x\log 5$ — 2 marks (1 mark for each theorem)

$x\log 3 + \log 3 = \log 4 + x\log 5$

$\log 3 - \log 4 = x\log 5 - x\log 3$

$\dfrac{\log 3 - \log 4}{\log 5 - \log 3} = x$ — 1 mark for simplification

$-0.563 = x$ — 1 mark for consistent answer

Note(s)

- final answer must be correct to at least 3 decimal places
- if brackets omitted in first step but assumed, **deduct** ½ mark
- values

  $\log 3 = 0.47712$ $\log 4 = 0.60205$ $\log 5 = 0.69897$

  $\ln 3 = 1.0986$ $\ln 4 = 1.3862$ $\ln 5 = 1.6094$

EXEMPLARS

3. Solve the equation $3^{x+1} = 4(5^x)$ algebraically.
   Express your final answer correct to 3 decimal places.

$$3^{x+1} = 4(5^x)$$
$$\log 3^{x+1} = \log 4(5^x)$$
$$x+1 \log 3 = x \log 4(5)$$
$$x\log 3 + \log 3 = x\log 20$$
$$-x\log 3 \qquad\qquad -x\log 3$$
$$\frac{\log 3}{\log 20 - \log 3} = \frac{x\log 20 - x\log 3}{\log 20 - \log 3}$$
$$x = 0{,}579$$

Mark:	**2½ out of 4**
Note(s):	• gave 1 mark for log theorem • gave 1 mark for simplification • gave 1 mark for consistent answer • deducted ½ mark for missing brackets in line 3

Question No. 4

Outcome: H3

Question

4. Evaluate: $\sum_{k=1}^{\infty} 2^{6-k}$.

3 marks

Solution

Method 1

$\sum_{k=1}^{\infty} 2^{6-k} = 32+16+8+....$ 1 mark for t_1

$S_\infty = \dfrac{32}{1-\frac{1}{2}}$ 1 mark for r; ½ mark for substitution

$= \dfrac{32}{\frac{1}{2}}$

$= 64$ ½ mark for final answer

Method 2

$2^{6-1} + 2^{6-2} + 2^{6-3} \ldots .$ 1 mark for expansion

$= \underbrace{2^5 + 2^4 + 2^3 + 2^2 + 2^1 + 2^0}_{63} + \underbrace{2^{-1} + 2^{-2} + 2^{-3} + \ldots .}_{1}$ ½ mark for 63; 1 mark for $2^{-1} + 2^{-2} + 2^{-3} + \ldots = 1$

$= 63 + 1$

$= 64$ ½ mark for final answer

Note(s)

- first step showing sigma expansion in Method 1 is not necessary

EXEMPLARS

4. Evaluate: $\sum_{k=1}^{\infty} 2^{6-k}$.

$$(2^{6-1}) + (2^{6-2}) + (2^{6-3}) \ldots (2^{6-\infty})$$

$$(2^5) + (2^4) + (2^3) + (2^2) + (2^1) + (2^0) \ldots (2^{6-\infty})$$

$$S_\infty = \frac{32}{1-2}$$

$$S_\infty = \frac{32}{-1}$$

$$S_\infty = -32$$

Mark: 2 out of 3
Note(s): • deducted 1 mark for incorrect r

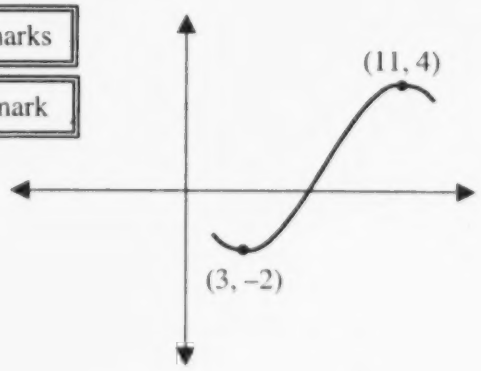
Question No. 5

Outcome: B7, A6

Question

5. A minimum and a maximum of a sinusoidal function are shown below.

a) Find an equation of this function. **4 marks**
b) Find the y-intercept. State the answer correct to 3 decimal places. **1 mark**



Solution

a) $y = -3\cos\left(\frac{\pi}{8}(x-3)\right)+1$ 4 marks (1 mark per parameter in the correct formula)

OR

$y = 3\sin\left(\frac{\pi}{8}(x-7)\right)+1$

OR

$y = 3\cos\left(\frac{\pi}{8}(x-11)\right)+1$ other equivalent equations may be given

b) y-intercept = – 0.148 1 mark { ½ mark for substitution of 0 for x; ½ mark for final answer }

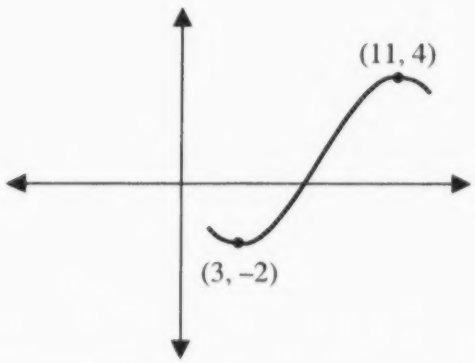
Note(s)

a)
- if answer not written as an equation, **deduct** ½ mark
- period = 16, amplitude = 3

b)
- give full mark if y-intercept is consistent with equation from a)
- **deduct** ½ mark for incorrect rounding
- final answer must be correct to at least 3 decimal places

EXEMPLARS

5. A minimum and a maximum of a sinusoidal function are shown below.



a) Find an equation of this function.

$A = 3$

$B = \frac{2\pi}{16} = \frac{\pi}{8}$

$C = -1$

$D = 1$

EQUATION $= -\sin\theta$

$3\sin\frac{\pi}{8}(0+1)+1$

$-3\sin\frac{\pi}{8}(0+1)+1$

Mark:	**2½ out of 4**
Note(s):	• gave 1 mark for A • gave 1 mark for B • gave 1 mark for D • deducted ½ mark for solution not written as an equation

b) Find the y-intercept. State the answer correct to 3 decimal places.

$X = 0$

$Y = -.15$

$-3\sin\frac{\pi}{8}(x+1)+1$

$-3\sin\frac{\pi}{8}(0+1)+1$

$-3\sin\frac{\pi}{8}(1)+1$

$-3(.38)(1)+1$

$(-1.15)+1$

$Y = 15$

Mark:	**½ out of 1**
Note(s):	• gave ½ mark for substitution of 0 for x

Question No. 6

Outcome: C2

Question

6. If $\cos\alpha = -\frac{3}{5}$ where $\tan\alpha > 0$ and $\sin\beta = \frac{5}{13}$ where $\sec\beta > 0$, find the exact value of $\sin(\alpha+\beta)$.

4 marks

Solution

$\sin\alpha = -\frac{4}{5}$ 1 mark for calculation

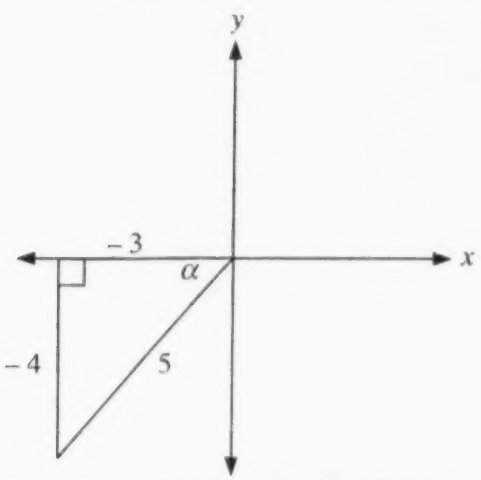
$\cos\beta = \frac{12}{13}$ 1 mark for calculation

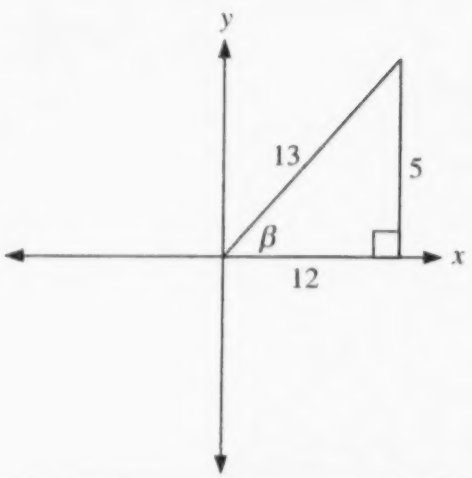
$\sin(\alpha+\beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$= \left(-\frac{4}{5}\right)\left(\frac{12}{13}\right) + \left(-\frac{3}{5}\right)\left(\frac{5}{13}\right)$ 1 mark

$= -\frac{48}{65} + -\frac{15}{65}$

$= -\frac{63}{65}$ 1 mark for final answer





Note(s)

- do not give ½ marks for line 1 or 2 if algebraic sign is incorrect because it is a conceptual error

EXEMPLARS

6. If $\cos\alpha = -\frac{3}{5}$ where $\tan\alpha > 0$ and $\sin\beta = \frac{5}{13}$ where $\sec\beta > 0$, find the exact value of $\sin(\alpha+\beta)$.

$\sin(\alpha+\beta) = \sin\alpha\cos\beta + -\frac{3}{5}\ \frac{5}{13}$

$\sin^2\beta = \left(\frac{5}{13}\right)^2 = \frac{25}{169}$

$\cos^2\alpha = \left(-\frac{3}{5}\right)^2 = \frac{9}{25}$

$\cos\beta = \sqrt{1-\frac{25}{164}} = 0.92$

$\sin\alpha = \sqrt{1-\frac{9}{25}} = 0.8$

$0.8 \cdot 0.92 + -\frac{3}{5} \cdot \frac{5}{13} = 0.51$

$\sin(\alpha+\beta) = 0.51$

Mark:	**2 out of 4**
Note(s):	• deducted ½ mark for rounding value of $\cos\beta$
	• deducted 1 mark for incorrect sign of $\sin\alpha$ (concept error)
	• deducted ½ for rounding final answer

Question No. 7

Outcome: E3

Question

7. In how many ways can 11 friends sit at two round tables if one table seats 6 and the other table seats 5?
   Briefly explain your calculations.

3 marks

Solution

$_{11}C_6$	•	5!	•	4!
choose 6 friends		arrange in circle first table		arrange in circle second table

$_{11}C_6 \cdot 5! \cdot 4!$
$= 462 \cdot 120 \cdot 24$
$= 1\,330\,560$

2 marks { 1 mark for combination; ½ mark for each permutation

1 mark for final answer

Note(s)

- deduct ½ mark if explanation of calculations is not given

EXEMPLARS

7. In how many ways can 11 friends sit at two round tables if one table seats 6 and the other table seats 5?
Briefly explain your calculations.

$_{11}C_6 \cdot 5! = 55440$

$_5C_5 \cdot 4! = 24$

$55440 + 24 = 55464$

∴ there are 55464 ways.

Mark:	**1½ out of 3**
Note(s):	• deducted 1 mark for adding instead of multiplying (concept error) • deducted ½ mark for no explanation

Question No. 8

Outcome: E3

Question

8. Ron is having a dinner party. He invites 7 out of his 10 friends.
   a) In how many ways can he choose his 7 guests? — 1 mark
   b) In how many ways can he choose his guests if two of them, Al and Sue, must attend together or not at all?
   Briefly explain your calculations. — 3 marks

Solution

a) $${}_{10}C_7 = \frac{10!}{7!3!} = 120$$ — 1 mark (½ mark for combination, ½ mark for answer)

b)

Method 1

Al and Sue are one group

If both attend: $${}_8C_5 = 56$$ — 1 mark for $${}_8C_5$$

If neither attends: $${}_8C_7 = 8$$ — 1 mark for $${}_8C_7$$

Total: $$= 64$$ — 1 mark for addition of cases

Method 2

Total ways all could attend $${}_{10}C_7 = 120$$ — (no marks for this line, since 1 mark is already given in a) for the same information)

Al attends and Sue doesn't $${}_8C_6 = 28$$ — ½ mark for combination

Sue attends and Al doesn't $${}_8C_6 = 28$$ — ½ mark for combination

Total ways $120-(28+28)=64$ — 2 marks for final answer { 1 mark for addition of cases; 1 mark for subtraction of cases }

Note(s)

- deduct ½ mark if identification of calculations is not given

EXEMPLARS

8. Ron is having a dinner party. He invites 7 out of his 10 friends.

   a) In how many ways can he choose his 7 guests?

$n = 10$
$r = 7$
nCr

$10C_7 = 180$ ways

Mark:	**½ out of 1**
Note(s):	• deducted ½ mark for incorrect calculation

   b) In how many ways can he choose his guests if two of them, Al and Sue, must attend together or not at all?
   Briefly explain your calculations.

$8C_5 = 56$

$10C_2 = 45$

$56 + 45 =$ 101 ways with them

$8C_7 = 8$ ways

Mark:	**1½ out of 3**
Note(s):	• gave 1 mark for ${}_8C_5$ • gave 1 mark for ${}_8C_7$ • gave 0 marks for addition of cases • deducted ½ mark for no identification of calculations

Question No. 9

Outcome: G3, G4

Question

9. In a Manitoba high school, 30% of the students are in Senior 2, 25% of the students are in Senior 3 and the rest of the students are in Senior 4.
   Francis ran for student president.
   The results were:
   40% of the Senior 2 students voted for her.
   20% of the Senior 3 students voted for her.
   60% of the Senior 4 students voted for her.

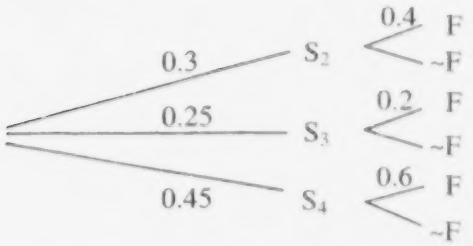
1 mark

   a) What is the probability that a randomly selected Senior 4 student did **not** vote for Francis?

   b) If a student voted for Francis, what is the probability that the student was in Senior 4?

3 marks

Solution



a) P(not Francis) = 1 – 0.6 = 0.4 — 1 mark

b) $P(S_4|\text{Francis}) = \dfrac{0.45 \times 0.6}{0.3 \times 0.4 + 0.25 \times 0.2 + 0.45 \times 0.6}$ — 2 marks { 1 mark for numerator; 1 mark for denominator }

$= \dfrac{0.27}{0.12 + 0.05 + 0.27}$

$= \dfrac{0.27}{0.44}$

$= \dfrac{27}{44} \quad (0.61\overline{36})$ — 1 mark for final answer

Note(s)

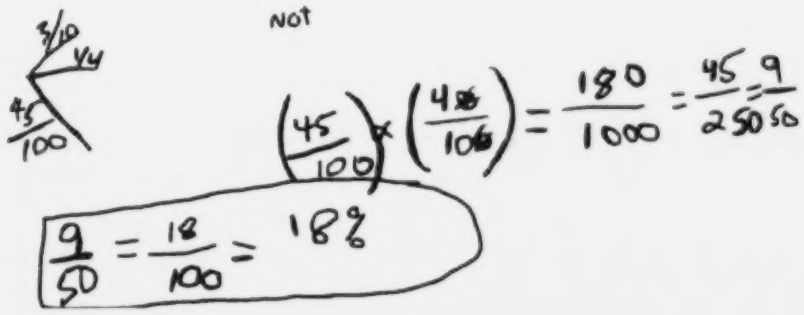
- give 1 mark if only a correctly labelled tree diagram is supplied

EXEMPLARS

9. In a Manitoba high school, 30% of the students are in Senior 2, 25% of the students are in Senior 3 and the rest of the students are in Senior 4.
Francis ran for student president.
The results were:
40% of the Senior 2 students voted for her.
20% of the Senior 3 students voted for her.
60% of the Senior 4 students voted for her.

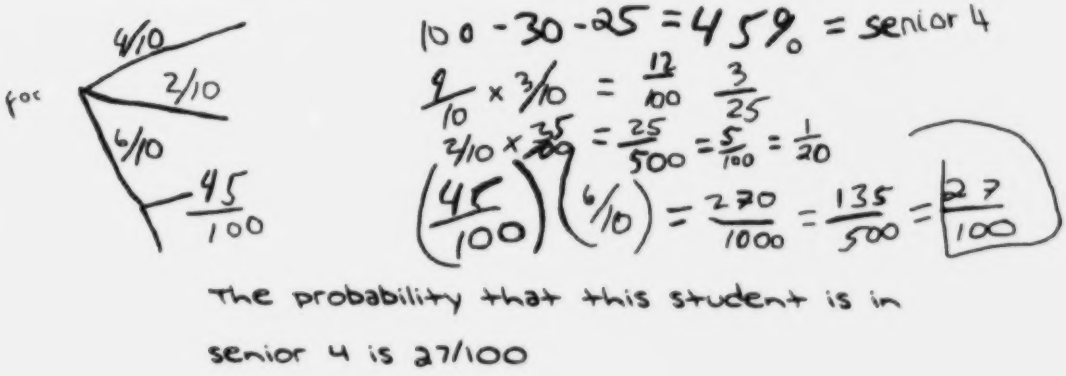
30% = S2
25% = S3
45% = S4

a) What is the probability that a randomly selected Senior 4 student did **not** vote for Francis?



Mark: **0 out of 1**
Note(s): • misinterpreted question

b) If a student voted for Francis, what is the probability that the student was in Senior 4?



Mark: **1 out of 3**
Note(s): • gave 1 mark for correct numerator

Question No. 10

Outcome: F3

Question

10. Write an equation of the circle which has the centre (2, –5) and is tangent to the x-axis.

2 marks

Solution

$r = 5$ 1 mark for radius

equation is $(x-2)^2 + (y+5)^2 = 25$

OR

$x^2 + y^2 - 4x + 10y + 4 = 0$

} 1 mark for equation

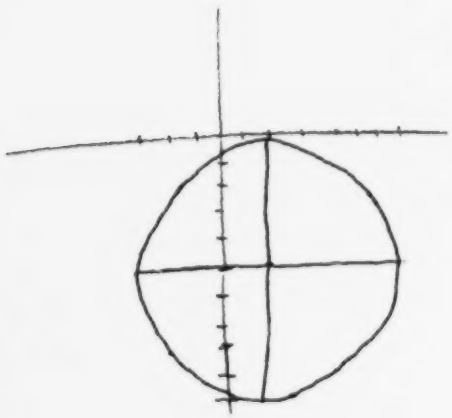
Note(s)

- radius need not be stated for full marks if correct equation given

EXEMPLARS

10. Write an equation of the circle which has the centre (2, –5) and is tangent to the x-axis.

$(x-2)(y+5) = 25$



Mark: **1 out of 2**
Note(s): • gave 1 mark for correct radius

Part 2—Restricted-Response Questions

Question No. 26

Outcome: A1

Question

26. Convert $\frac{7\pi}{18}$ to degrees. Simplify your answer.

1 mark

Answer

$\frac{7\pi}{18}$

$= \frac{7 \times 180°}{18}$

$= 70°$

OR

$= \frac{7\pi}{18}$

$= \frac{7\pi}{18} \cdot \frac{180°}{\pi}$

$= 70°$

½ mark

Note(s)

- units need not be included since they are stated in the question

Question No. 27

Outcome: A2

Question

27. Give a possible value for $\sin\theta$ if $\cos\theta = \frac{1}{\sqrt{10}}$.

1 mark

Answer

Method 1

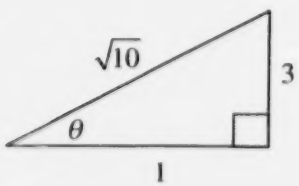
$\cos^2\theta + \sin^2\theta = 1$

$\left(\frac{1}{\sqrt{10}}\right)^2 + \sin^2\theta = 1$ ½ mark

$\sin^2\theta = \frac{9}{10}$

$\sin\theta = \pm\frac{3}{\sqrt{10}}$ OR $\pm\frac{3\sqrt{10}}{10}$ OR $\frac{3}{\sqrt{10}}$ OR $-\frac{3}{\sqrt{10}}$

Method 2



½ mark for diagram

$\sin\theta = \frac{3}{\sqrt{10}}$

Question No. 28

Outcome: C1

Question

28. Simplify $\cos^2(\alpha+\beta)+\sin^2(\alpha+\beta)$.

1 mark

Answer

$=1$

Note(s)

- no ½ marks

Question No. 29

Outcome: A6, B7

Question

29. What is the period of $g(x) = 2\sin(3x) - 1$?

1 mark

Answer

$$\text{Period} = \frac{2\pi}{|b|} = \frac{2\pi}{3}$$

Question No. 30

Outcome: B3

Question

30. The coordinates of a point on the graph of $y = h(x)$ are (–3, 5).
    State the coordinates of a point on the graph of $y = h(-x)$.

1 mark

Answer

(3,5)

Note(s)

- give ½ mark if student identifies a reflection over the y-axis without coordinates

Question No. 31

Outcome: D5

Question

31. If $\log_a 2 = p$ and $\log_a 3 = q$, find an expression for $\log_a 6$ in terms of p and q.

1 mark

Answer

$\log_a 6$

$= \log_a (2 \times 3)$

$= (\log_a 2 + \log_a 3)$

$= (p + q)$ OR $p + q$

Note(s)

- give ½ mark for correct log theorem on line 3

Question No. 32

Outcome: F1

Question

32. Identify the conic represented by the equation: $x^2 - 4x = y^2 + 2$. 1 mark

Answer

hyperbola

Question No. 33

Outcome: H1

Question

33. A geometric sequence has $t_2 = 100$ and $t_4 = 4$.
Find a possible value of the common ratio, r.

1 mark

Answer

Method 1

$t_2 = t_1 r = 100$

$t_4 = t_1 r^3 = 4$

$r^2 = \frac{4}{100}$

$r = \pm\sqrt{\frac{4}{100}}$

$r = \pm\frac{1}{5}$ OR $r = \frac{1}{5}$ OR $r = -\frac{1}{5}$

Method 2

$t_4 = t_2 r^2$

$4 = 100r^2$

$r^2 = \frac{4}{100}$

$r = \pm\sqrt{\frac{4}{100}}$

Note(s)

- give ½ mark for $r^2 = \frac{4}{100}$ or $4 = 100r^2$

Question No. 34

Outcome: E2

Question

34. How many five-digit arrangements are possible using all of the digits 1, 2, 3, 4, 5, if the middle digit must be odd and repetition of digits is not allowed?

1 mark

Answer

$\underline{4} \times \underline{3} \times \underbrace{\underline{3}}_{\text{Odd}} \times \underline{2} \times \underline{1}$ OR $4!\,3$ OR 72 OR $5! - 2 \cdot 4!$

Question No. 35

Outcome: G3

Question

35. The probability of hitting a target is $\frac{1}{3}$. Therefore, the probability of missing on 3 consecutive tries is $\frac{8}{27}$.

    What is the probability of hitting this target at least once in 3 tries?

1 mark

Answer

$$P(\text{at least 1 hit}) = 1 - \frac{8}{27} = \frac{19}{27}$$

Question No. 36

Outcome: F3

Question

36. State the coordinates of the vertex of the relation $y^2 = 4x + 4$.

1 mark

Answer

$y^2 = 4x + 4$

$y^2 = 4(x + 1)$

vertex = (–1, 0)

Note(s)

- give ½ mark for correct graph with vertex not given as a coordinate pair

Question No. 37

Outcome: A3

Question

37. If $\sin(5\theta) = \frac{\sqrt{3}}{2}$, where 5θ is an acute angle, find the exact value of θ. 1 mark

Answer

$\sin(5\theta) = \frac{\sqrt{3}}{2}$

$5\theta = 60°$ ½ mark

$\theta = 12°$

OR

$5\theta = \frac{\pi}{3}$ ½ mark

$\theta = \frac{\pi}{15}$

Note(s)

- give ½ mark for $\theta = 24°$ or $\frac{2\pi}{15}$

Question No. 38

Outcome: D7

Question

38. Find $f^{-1}(x)$, if $f(x)=e^x$.

1 mark

Answer

$f(x)=e^x$

$f^{-1}(x)=\ln x$ OR $f^{-1}(x)=\log_e x$

OR

$f^{-1}(x)=\dfrac{\log x}{\log e}$

Note(s)

- give ½ mark for $x=e^y$

Question No. 39

Outcome: G3

Question

39. A box contains 2 red, 3 black and 2 white balls. A ball is selected at random from the box. What is the probability that the ball is **not** white?

1 mark

Answer

$$P(\text{Not White}) = \frac{5}{7}$$

OR

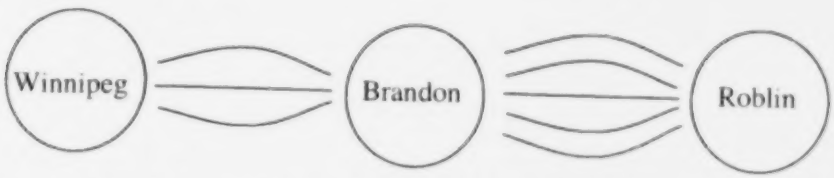
$$P(\text{Not White}) = 1 - \frac{2}{7}$$

Question No. 40

Outcome: E1

Question

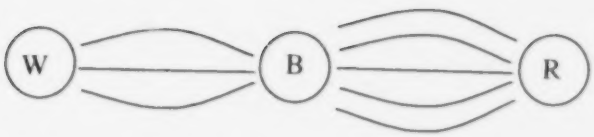
40.



There are 3 roads between Winnipeg and Brandon and 5 roads between Brandon and Roblin. How many different routes are there from Winnipeg to Roblin through Brandon?

1 mark

Answer



$3 \times 5 = 15$

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Part 2—Open-Response Questions

Question No. 41

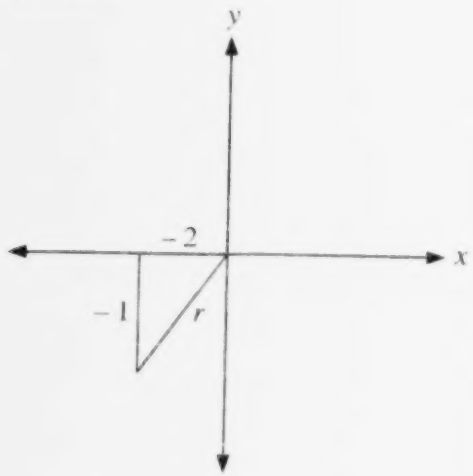
Outcome: A2

Question

41. Find the coordinates of the point of intersection of the unit circle with the line segment joining the origin to the point (–2, –1).

3 marks

Solution



$r^2 = (-2)^2 + (-1)^2$

$r^2 = 4 + 1$

$r = \sqrt{5}$ — 1 mark for value r

pt. of intersection is

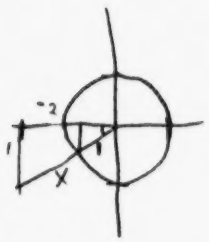
$\left(-\frac{2}{\sqrt{5}}, -\frac{1}{\sqrt{5}}\right)$ — 1 mark for value x-coordinate; 1 mark for value y-coordinate

Note(s)

- $x = \frac{-2}{\sqrt{5}}$ $y = \frac{-1}{\sqrt{5}}$ is acceptable
- ½ mark deduction for each sign error

EXEMPLARS

41. Find the coordinates of the point of intersection of the unit circle with the line segment joining the origin to the point (–2, –1).



$a^2+b^2=c^2$
$2^2+1^2=c^2$
$5=c^2$
$c=\sqrt{5}$

$\frac{1}{\sqrt{5}} = \frac{x}{2}$

$-\frac{2}{2\sqrt{5}}, -\frac{1}{\sqrt{5}}$

$\frac{1}{\sqrt{5}} = \frac{y}{1}$

Mark: **1½ out of 3**
Note(s):
- gave 1 mark for the correct r value
- gave 1 mark for y-value
- deducted ½ mark for the missing brackets

Question No. 42

Outcome: F2

Question

42. Change the following equation to standard form:

$$5y^2 + 50y - 4x^2 + 24x + 69 = 0$$

3 marks

Solution

$$5y^2 + 50y - 4x^2 + 24x = -69$$

$$5\left(y^2 + 10y\right) - 4\left(x^2 - 6x\right) = -69$$

$$5\left(y^2 + 10y + 25\right) - 4\left(x^2 - 6x + 9\right) = -69 + 5(25) - 4(9)$$

1 mark for completing square for y

1 mark for completing square for x

$$5(y+5)^2 - 4(x-3)^2 = 20$$

$$\frac{5(y+5)^2}{20} - \frac{4(x-3)^2}{20} = \frac{20}{20}$$

$$\frac{(y+5)^2}{4} - \frac{(x-3)^2}{5} = 1$$

1 mark for final standard form

EXEMPLARS

42. Change the following equation to standard form:

$$5y^2 + 50y - 4x^2 + 24x + 69 = 0$$

$-4x^2+5y^2+24x+50y+69=0$

$(-4x^2+24x)+(5y^2+50y)=-69$

$-4-4(x-6x+9)+5(y^2+10y+25)=-69-36+125$

$=-4(x-3)^2+5(y+5)^2=+20$

$\left(\frac{-6}{2}\right)^2=9$

$\left(\frac{+10}{2}\right)^2=25$

$9\times4=36$

$5\times25=125$

$\begin{array}{r} {\scriptstyle 1}\\ 69\\ +36\\ \hline 105 \end{array}$

Mark:	**1½ out of 3**
Note(s):	• gave 1 mark each for completing the square for x and y • deducted ½ mark for notation errors on line 3

Question No. 43

Outcome: D6

Question

43. Solve for x:

3 marks

$$\log_2(\log_{81} x) = -2$$

Solution

$2^{-2} = \log_{81} x$	1 mark for exponential form
$\frac{1}{4} = \log_{81} x$	½ mark for evaluating 2^{-2}
$81^{\frac{1}{4}} = x$	1 mark for exponential form
$3 = x$	½ mark for answer

EXEMPLARS

43. Solve for x:

$$\log_2(\log_{81} x) = -2$$

$2^{-2} = \log_{81} x$ $\qquad 2^{-2} \cdot 4$

$81^{2^{-2}} = x$

$81^{-4} = x$

$81^{4} = x$

$x = 3$

$3 \cdot 3 = 9 \cdot 3 = 27 \cdot 3$

27

$\cdot 3$

$\overline{81}$

Mark:	**2 out of 3**
Note(s):	• two correct conversions to exponential form

Question No. 44

Outcome: B1, B3

Question

44. Given the function $f(x)=\sqrt{x+4}-3$, find:

a) x-intercept of $f(x)$ — 1 mark

b) domain of $f(x)$ — 1 mark

c) x-intercept of $f^{-1}(x)$ — 1 mark

d) domain of $f^{-1}(x)$ — 1 mark

Solution

a) x-intercept of $f(x)$ is 5 — 1 mark { ½ mark for substituting 0 for y; ½ mark for evaluating }

b) domain of $f(x)$ is $x \geq -4$ OR $[-4, \infty)$ — 1 mark { ½ mark for $x+4 \geq 0$; ½ mark for answer }

c) x-intercept of $f^{-1}(x)$ is $x=\sqrt{0+4}-3$

$x=2-3$

$x=-1$

1 mark { ½ mark for $\sqrt{0+4}-3$; ½ mark for answer }

d) domain of $f^{-1}(x)$ is range of $f(x)$ — 1 mark

$x \geq -3$ OR $[-3, \infty)$

Note(s)

- if x-intercept given as coordinates of a point, **deduct** ½ mark only once
- award ½ mark for part b) $(-4, \infty)$, $[-4, \infty]$, or $x > -4$
- award ½ mark for part d) $(-3, \infty)$, $[-3, \infty]$, or $x > -3$

EXEMPLARS

44. Given the function $f(x) = \sqrt{x+4} - 3$ find:

a) x-intercept of $f(x)$ 5

$0 = \sqrt{x+4} - 3$

$+3 \quad +3$

$3 = \sqrt{x+4}$

$3^2 = (\sqrt{x+4})^2$

$9 = x+4$

$-4 \quad -4$

$5 = x$

Mark: 1 out of 1

b) domain of $f(x)$ $y > -3$

Mark: 0 out of 1

c) x-intercept of $f^{-1}(x)$ 1

$x = \sqrt{y+4} - 3$

$x = \sqrt{0+4} - 3$

$x = \sqrt{4} - 3$

$x = 1$

Mark: ½ out of 1 **Note(s):** • work shown, numerical error

d) domain of $f^{-1}(x)$ $y > -3$

Mark: 0 out of 1 **Note(s):** • ½ mark deducted for use of y instead of x • ½ mark deducted for missing inclusion of –3 (i.e., $y \geq -3$)

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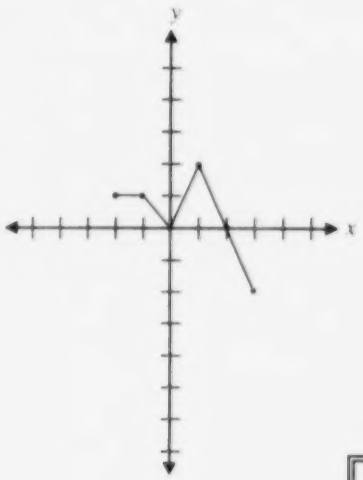
Question No. 45

Outcome: B1, B3, B5

Question

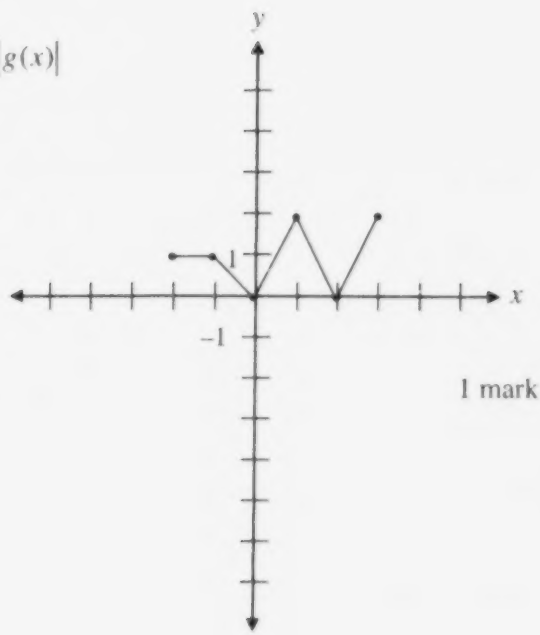
45. The graph of $y = g(x)$ is given below.



Solution

Sketch:

a) $y = |g(x)|$



1 mark

1 mark for segment from (2, 0) to (3, 2)

Note(s)

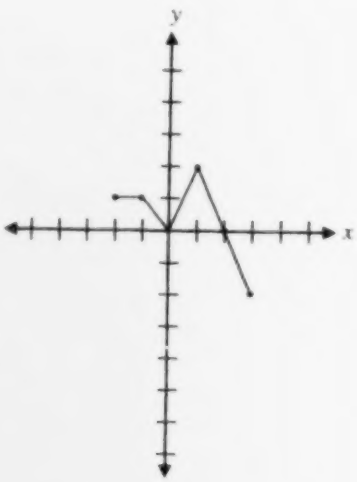
- deduct ½ mark if one or more arrows are put on graphs of a, b, or c

Question No. 45

Outcome: B1, B3, B5

Question

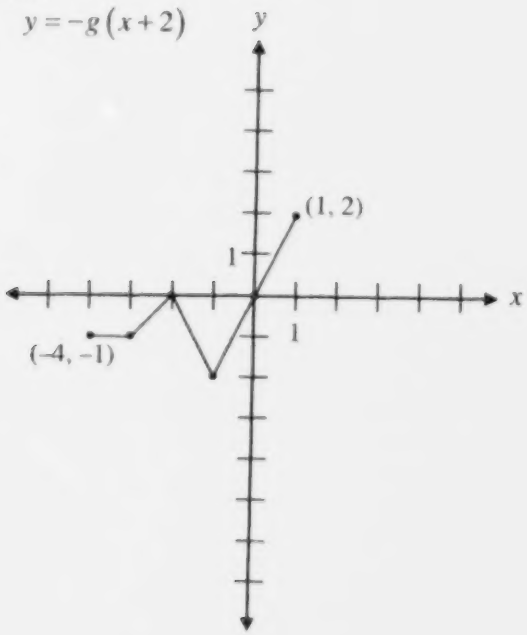
45. The graph of $y = g(x)$ is given below.



Solution

Sketch:

b) $y = -g(x+2)$



2 marks

1 mark for vertical reflection over x-axis

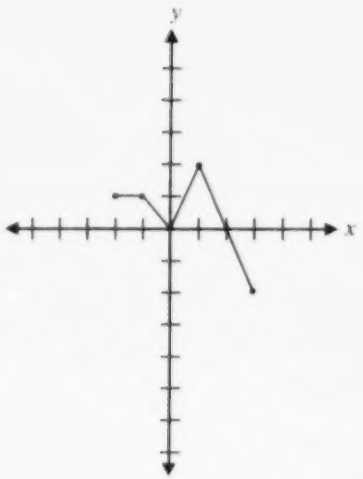
1 mark for horizontal translation left 2

Question No. 45

Outcome: B1, B3, B5

Question

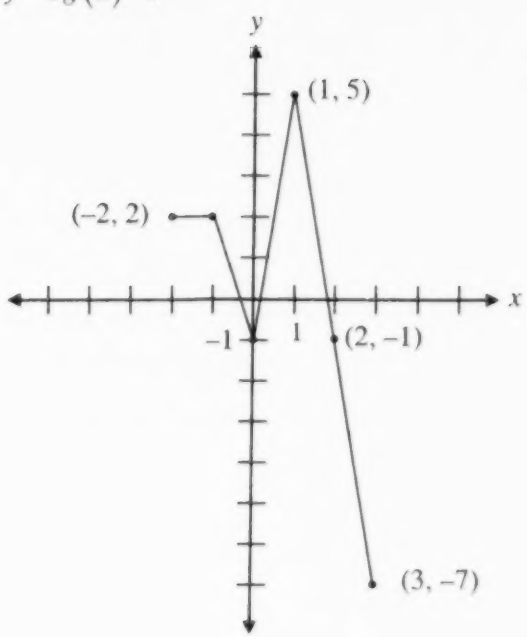
45. The graph of $y = g(x)$ is given below.



Solution

Sketch:

c) $y = 3g(x) - 1$



2 marks

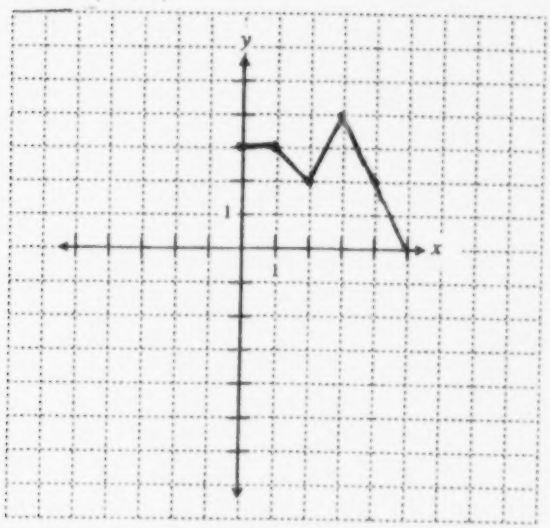
1 mark for vertical stretch

1 mark for vertical translation down 1

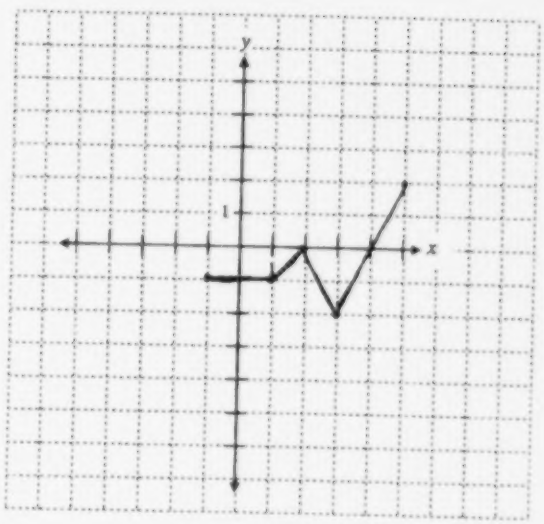
EXEMPLARS

45. The graph of $y = g(x)$ is given below.
Sketch:

a) $y = |g(x)|$



b) $y = -g(x+2)$



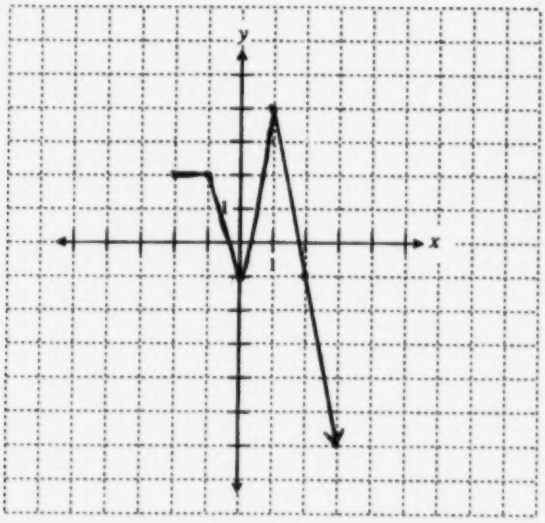
a)	**Mark:** **0 out of 1**

b)	**Mark:** **½ out of 2** **Note(s):** • gave 1 for reflection over x-axis • gave 0 for translation right 2 • deducted ½ mark for extra piece drawn $(0, -1)$ to $(-1, -1)$

EXEMPLARS

45. The graph of $y = g(x)$ is given below.
Sketch:

c) $y = 3g(x) - 1$



Mark:	**1½ out of 2**
Note(s):	• deducted ½ mark for arrow instead of endpoint

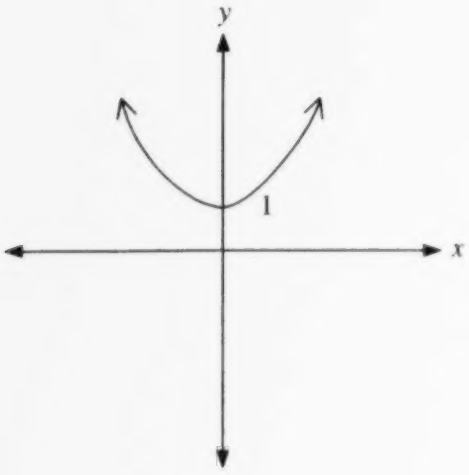
Question No. 46

Outcome: B4

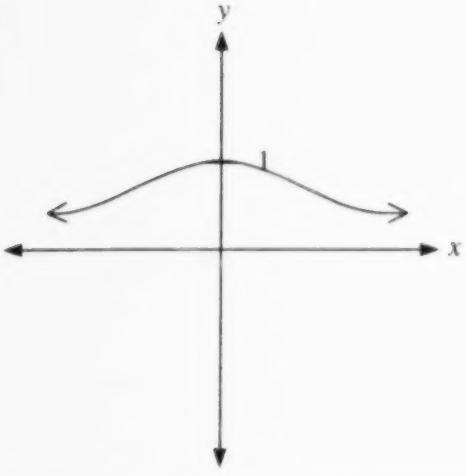
Question

46. The graph of $f(x) = x^2 + 1$ is given below. Sketch the graph of $\frac{1}{f(x)}$.

2 marks



Solution



1 mark for maximum point (0, 1)

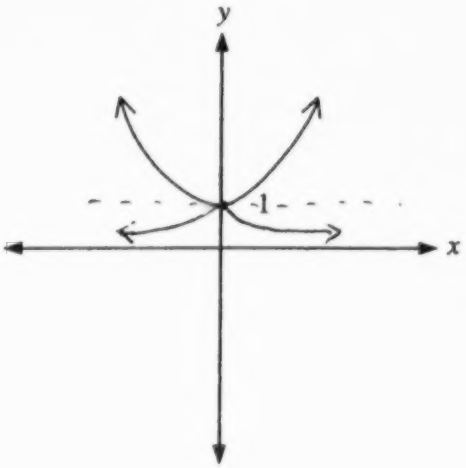
1 mark for asymptotic behaviour of the curve

Note(s)

- deduct maximum ½ mark for graphing errors such as missing arrows or curving away from x-axis

EXEMPLARS

46. The graph of $f(x) = x^2 + 1$ is given below. Sketch the graph of $\frac{1}{f(x)}$.



Mark:	**2 out of 2**
Note(s):	• gave full marks although graph shows cusp

Question No. 47

Outcome: C2

Question

47. Find the exact value of $\tan(75°)$.

3 marks

Solution

$75° = 45° + 30°$ — 1 mark for $75° = 45° + 30°$ or $(120° - 45°)$

$$\tan(75°) = \tan(45° + 30°)$$

$$= \frac{\tan 45° + \tan 30°}{1 - \tan 45° \tan 30°}$$ — ½ mark for correct identity

$$= \frac{1 + \frac{1}{\sqrt{3}}}{1 - (1)\left(\frac{1}{\sqrt{3}}\right)}$$ — ½ mark for $\tan 45° = 1$; ½ mark for $\tan 30° = \frac{1}{\sqrt{3}}$

$$= \frac{\sqrt{3} + 1}{\sqrt{3} - 1}$$ — ½ mark

OR

$$= 2 + \sqrt{3}$$

Note(s)

- deduct ½ mark for rationalizing denominator incorrectly
- other combinations for 75° are acceptable
- $75° = \frac{5\pi}{12}$

EXEMPLARS

47. Find the exact value of $\tan(75°)$.

$$75 \cdot \frac{\pi}{180} = \frac{75\pi}{180} = \frac{25\pi}{60} = \frac{5\pi}{12}$$

$$\frac{2\pi}{3} - \frac{\pi}{4} = \frac{5\pi}{12}$$

$$\tan(a - B) = \frac{\tan a - \tan B}{1 + \tan a \tan B}$$

$$= \frac{\frac{\sqrt{2}}{2} - 1}{1 + \frac{\sqrt{2}}{2}}$$

$$\tan 75° = \frac{\frac{\sqrt{2}}{2} - 1}{1 + \frac{\sqrt{2}}{2}}$$

Mark: **2½ out of 3**

Note(s): • deducted ½ mark for incorrect function value $\left(\tan\frac{2\pi}{3}\right)$

Question No. 48

Outcome: C2

Question

48. Prove the following identity:

$$\frac{1-\tan^2\theta}{1+\tan^2\theta} = \cos 2\theta$$

3 marks

Solution

Method 1

LHS

$= \dfrac{1-\tan^2\theta}{\sec^2\theta}$	1 mark for substitution of identity
$= \left(1-\tan^2\theta\right)\cos^2\theta$	½ mark for change to $\cos^2\theta$
$= \left(1-\dfrac{\sin^2\theta}{\cos^2\theta}\right)\cos^2\theta$	½ mark
$= \cos^2\theta - \sin^2\theta$	½ mark
$= \cos 2\theta$	½ mark

Method 2

LHS

$= \dfrac{\left(1-\dfrac{\sin^2\theta}{\cos^2\theta}\right)\cos^2\theta}{\left(1+\dfrac{\sin^2\theta}{\cos^2\theta}\right)\cos^2\theta}$	1 mark
$= \dfrac{\cos^2\theta-\sin^2\theta}{\cos^2\theta+\sin^2\theta}$	1 mark
$= \cos^2\theta - \sin^2\theta$	½ mark
$= \cos 2\theta$	½ mark

Method 3

LHS

$= \dfrac{1-\dfrac{\sin^2\theta}{\cos^2\theta}}{1+\dfrac{\sin^2\theta}{\cos^2\theta}}$	½ mark
$= \dfrac{\dfrac{\cos^2\theta-\sin^2\theta}{\cos^2\theta}}{\dfrac{\cos^2\theta+\sin^2\theta}{\cos^2\theta}}$	½ mark
$= \dfrac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin^2\theta}$	1 mark
$= \cos^2\theta - \sin^2\theta$	½ mark
$= \cos 2\theta$	½ mark

Note(s)

- **deduct** maximum ½ mark for θ missing two or more times

EXEMPLARS

48. Prove the following identity:

$$\frac{1-\tan^2\theta}{1+\tan^2\theta} = \cos 2\theta$$

$$\begin{aligned}
LHS &= \frac{1-\tan^2\theta}{1+\tan^2\theta} \\
&= \frac{1-\tan^2\theta}{\sec^2\theta} \\
&= \frac{1-\tan^2\theta}{\frac{1}{\cos^2\theta}} \\
&= 1-\frac{\sin^2\theta}{\cos^2\theta}\cdot\frac{\cos^2\theta}{1} \\
&= 1-\sin^2\theta \\
&= \cos^2\theta
\end{aligned}$$

Mark: **1½ out of 3**
Note(s): • brackets were omitted in "line 4" leading to an incorrect conclusion

Question No. 49

Outcome: D2

Question

49. Solve for x:

$$\left(\frac{1}{4}\right)^{2x-1} = 8^{x}$$

2 marks

Solution

$\left(2^{-2}\right)^{2x-1} = \left(2^{3}\right)^{x}$ } ½ mark for 2^{-2}, ½ mark for 2^{3}

$2^{-4x+2} = 2^{3x}$

$-4x + 2 = 3x$ ½ mark for equating exponents

$2 = 7x$

$\frac{2}{7} = x$ ½ mark

EXEMPLARS

49. Solve for x:

$$\left(\frac{1}{4}\right)^{2x-1} = 8^x$$

$\left(\frac{1}{2}\right)^{2(2x-1)} = (2^3)^x$

$2^{-2(2x-1)} = 2^{3x}$

$-2(2x-1) = 3x$

$-4x + 2 = 3x$

$2 = 10x$

$\frac{2}{10} = x$

Check ✓

The value of $x = \frac{1}{5}$

Mark:	**1½ out of 2**
Note(s):	• deducted ½ mark for error in exponent in line 2

Question No. 50

Outcome: A4

Question

50. Solve for x, where $0 \le x \le 2\pi$:

$$\sec^2 x - \sec x = 2$$

4 marks

Solution

Method 1

$\sec^2 x - \sec x - 2 = 0$		
$(\sec x - 2)(\sec x + 1) = 0$		½ mark for factoring
$\sec x = 2$	$\sec x = -1$	1 mark (½ mark for each solution)
$\cos x = \frac{1}{2}$	$\cos x = -1$	1 mark (½ mark for each solution)
$x = \frac{\pi}{3}, \frac{5\pi}{3}$	$x = \pi$	1½ marks (½ mark per solution)

Method 2

$\sec^2 x - \sec x = 2$		
$\frac{1}{\cos^2 x} - \frac{1}{\cos x} = 2$		½ mark for change to $\cos x$
$1 - \cos x = 2\cos^2 x$		½ mark for clearing denominators
$2\cos^2 x + \cos x - 1 = 0$		
$(2\cos x - 1)(\cos x + 1) = 0$		½ mark
$\cos x = \frac{1}{2}$	$\cos x = -1$	1 mark
$x = \frac{\pi}{3}, \frac{5\pi}{3}$	$x = \pi$	1½ marks

Note(s)

- maximum of 3 marks for solutions in degrees

EXEMPLARS

50. Solve for x, where $0 \le x \le 2\pi$:

$\sec^2 x - \sec x = 2$

$\sec^2 x - \sec x = 2$

$\sec x(\sec x - 1) = 2$

$\sec x = 2 \qquad \sec x - 1 = 2$

$\cos x = \frac{1}{2} \qquad \cancel{\sec x = 3}$ impossible

$x = \frac{\pi}{3}, \frac{2\pi}{3}$

Mark: **1½ out of 4**

Note(s):
- gave ½ mark for factoring
- gave ½ mark for $\sec x = 2 \rightarrow \cos x = \frac{1}{2}$
- gave ½ mark for $\frac{\pi}{3}$ solution

Question No. 51

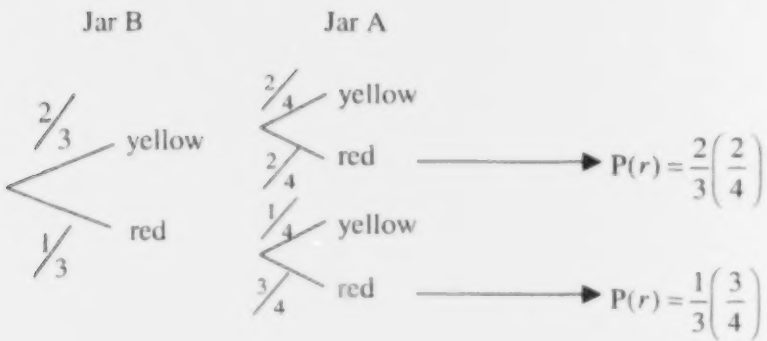
Outcome: G2

Question

51. Jar A contains 1 yellow and 2 red marbles.
Jar B contains 2 yellow and 1 red marbles.
A marble is chosen at random from Jar B and placed in Jar A.
Find the probability of now choosing a red marble from Jar A.

3 marks

Solution



$$P(r \text{ from Jar A}) = \frac{2}{3}\left(\frac{2}{4}\right) + \frac{1}{3}\left(\frac{3}{4}\right)$$ 2½ marks (½ for each fraction, ½ for addition)

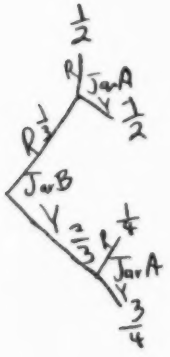
$$= \frac{1}{3} + \frac{1}{4}$$
$$= \frac{7}{12}$$ } ½ mark for answer

Note(s)

- the tree diagram is optional
- if only a correct tree diagram given, give 1 mark

EXEMPLARS

51. Jar A contains 1 yellow and 2 red marbles.
Jar B contains 2 yellow and 1 red marbles.
A marble is chosen at random from Jar B and placed in Jar A.
Find the probability of now choosing a red marble from Jar A.



$P(Red) = P(Red|Y1) + P(R|R)$

$P(Red) = \left(\frac{2}{3} \cdot \frac{1}{4}\right) + \left(\frac{1}{3} \cdot \frac{1}{2}\right)$

$P(Red) = \frac{1}{6} + \frac{1}{6}$

$P(Red) = \frac{1}{3}$

Mark:	**2 out of 3**
Note(s):	• deducted 1 mark for 2 incorrect fractions